

Why Quantum Computing does not Offer a Computational Speedup When Performing Addition

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Abstract— Quantum computing is used to solve complex computational problems. This solutions are based on addition, therefore addition is so often performed by a computer, although it is a simple to compute task, it is questioned whether a quantum computer can perform addition faster than its classical counterpart. The finding of this paper is: Classical and Quantum addition are both linear in performance. Quantum computation can be more efficient through a paradigm shift based on the quantum phenomena of state discrimination/distinguishability to computer with a higher number base.

Keywords. Finite state adder, Computational complexity, Quantum computing

I. DEFINITIONS

Quantum computer: Computing device that operates based on the science of quantum mechanics.

Computational task: Computation specified by the user of a computer that provides a result at the end and can be broken down into operational steps performable by a computer.

Qubit: Smallest unit in a quantum computer used to perform computation. A Qubit in the quantum paradigm is equal to the classical bit in the classical paradigm with the difference that the quantum bit can be both zero and one simultaneously, while the classical bit is either one or the other.

II. INTRODUCTION

Attention in computer science is gradually and steadily moving towards quantum computing. Although the quantum computer is currently not achievable, since researchers demonstrated that it could be possible to speed up the computational performance of some hard to solve problems, the amount of research into developing a quantum

computer has been almost limitless. While the research on quantum computing is demonstrated that it could be possible to solve some problems more efficiently, however, currently it cannot be generalised to all computational problems [1-3]. In particular, this paper suggests that addition is not performed faster through a quantum computer. Even though addition is not a hard to solve computational task, the very fact that it is performed so often by a computer means that improving its performance is important. This would be insignificant for a single computation, but could improve overall computation performance over a period of time.

III. BACKGROUND

In principle, quantum computing and classical computing are the same. However, it was demonstrated in theory that quantum computing would be able to solve some computational problems in less time than in classical computing. Deutsch who confirmed through demonstration the prior speculation that quantum mechanics enabled an increased computational speedup [4], based on the idea of the multi-universe interpretation of quantum mechanics [5]. The quantum computing is different from classical computing at its most fundamental level, the 'Bit' or in the case of a quantum computer the 'Qubit' [6]. While the classical 'Bit' can hold either the values *one* or *zero*, the 'Qubit' is able to hold both in the same time. This scales as more 'Qubits' are included such that when two 'Qubits' are used four values between *zero* and *four* can be stored, in the case of *three* values this becomes *eight*. Each of those values is stored in a universe. A further advantage is that this enables computation to be performed on all values at the same time; however at the end of a computation only *one* result can be retrieved. This is known as the *quantum parallelism thesis*, which is believed to be the key reason why quantum computers are faster than classical computers [7]. For this research the authors suggest an

alternative approach to viewing the quantum computing paradigm. This is based on state discrimination[8-11]/distinguishability[12] through which a model of computation with a higher radix can be represented.

IV. STATEMENT OF MAIN RESULTS

This paper presents the idea that the principles of quantum mechanics used to perform computation is not improved the performance of computing addition. This is based on the evaluation and comparison of the logic gates used in the classical paradigm to perform addition and compared against the logic gates used in the quantum paradigm to perform addition. Furthermore, a finite state adder is presented that can perform the computational task *addition* equally for both the paradigms. Therefore, it can be concluded that the performance of a classical computer is no different than the quantum, except for very specific computational problems. The computational performance of an adder in both the quantum and classical paradigm is demonstrated to be of linear computational complexity. Through an alternative view to the quantum computing paradigm it was found that the same computational task can be computed in less steps.

V. METHODOLOGY

The process of evaluating the computational performance of the computational *task addition* in classical and quantum computing is based on the realist ontological perception of computation. As a means of comparing classical addition against quantum addition, the circuit model of computation is adopted from the literature. The evaluation of its computational performance is achieved through a finite state adder model of computation, which is the same for both the classical and quantum paradigm. The performance is accessed through computational complexity theory. The complexity class computational task of addition can be determined in the classical and quantum paradigm to be determined.

VI. PROOF

The proof that demonstrates computation of addition to be equal in performance in either paradigm consists of two parts as flowing: there are the two circuit diagrams presented below. Figure one [13] demonstrates the classical half adder circuit, while figure two [14] demonstrates the quantum half adder circuit. Both of these circuits have *two* logic gates. However, in the classical circuit there are only *two* inputs and *two*

outputs, while in the quantum circuit there are three. In addition, the operations of the logic gates are different, as they operate on different types of 'Bits' (Qubits in the quantum paradigm).

Circuit models of computation

In the classical half adder circuit the 'Exclusive OR' logic gate outputs a *one* if either of the *two* inputs is *one* and a *zero*. The *second logic gate* performs the operation that represents the carry of the arithmetical operation. Through an 'and' logic gate it is able to output *one* when both inputs are *one*. In summary, the outputs for this circuit are *zero* only when both inputs are *zero*. The sum is *one* and the carry *zero* when *one* of the *two* inputs is *zero*. Lastly, the sum is *zero* and the carry is *one* when both the inputs are *one*. The half adder circuit represents the most fundamental building block for computers to perform addition.

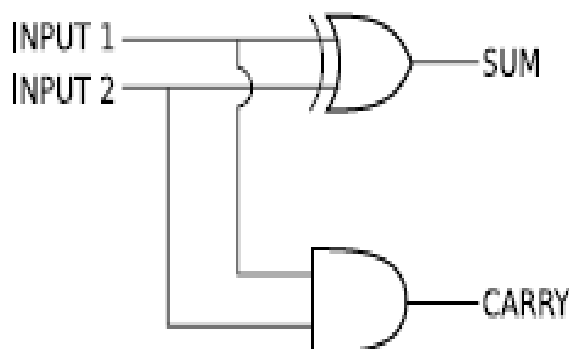


FIGURE I CLASSICAL ADDER

Figure two represents the equivalent of the classical half adder circuit in the quantum paradigm. A quantum half adder circuit also consists out of two logic gates which are the 'Toffoli gate' and the 'C-not logic gate'. The 'Toffoli gate' performs the same function as the 'and' gate in the classical paradigm as it determines if there will be a carry based upon both inputs being *one*. If this is not the case then the carry output remains *zero* and the 'C-not logic gate' is performed. In the event that the control 'Qubit' is *zero* and the target Qubit *one*, then the target Qubit remains and the sum is *one*. Alternatively the control Qubit is *one* and the target Qubit gets flipped such that the sum is *one*. In the case that both Qubits are *one* then the carry would be *one* and therefore the control Qubit would set the output of the sum to *zero*, equally the sum

would be zero when both inputs are zero

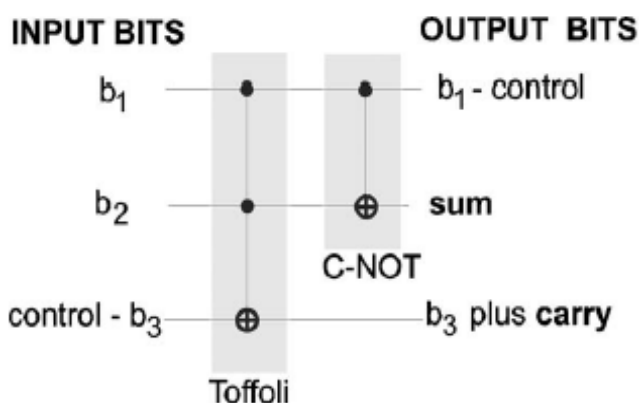


FIGURE II QUANTUM ADDER

Finite automaton model of computation

When defining the computational task of addition through a finite state adder as illustrated in figure three, the finite state adder model of computation can be used to perform computation of addition for both the classical half adder circuit and the quantum half adder circuit. This demonstrates that the finite state adder automaton model of computation is the same for both paradigms. Furthermore, the evaluation of the performance is that they are equal and both have a linear computational complexity when performing the computational task of addition.

The model of computation consists of five states and starts at the first state. Each operation causes a transition in state (operations can occur that causes the same state to be maintained). The transition is defined by the input. Based upon what the output is the transition occurs (the output is indicated after the forward slash of each state). The arrows between the states indicate what the inputs are. This is either '0/0', '01/10' or '11'. As the amount of input is linearly proportional to the outputs the computational complexity is of linear order. A computation of "6 + 23 = 29" (binary equivalent: "00110 + 10111 = 11101") is demonstrated in table 1.

TABLE I

Computation step	Computational state	Input	Carry	output	Result
Start	1				
1	3	0+1	0	1	1
2	4	1+1	1	01	1
3	5	1+1	1	101	5
4	3	0+0	0	1101	13
5	3	0+1	0	11101	29

In table 2, an evaluation has been demonstrated which considers the same computational task (6 + 23), with the difference that during this evaluation the arithmetic was based on a decimal finite state automaton model of computation (figure 2 part 1-2). This demonstrated that the computation would be performed in less than half the amount of operations. Based on those initial findings it is questioned whether there is already a more efficient method developed in the literature to perform addition, and which implication there are for computing with a model of computation that has a decimal radix.

Decimal finite state adder

TABLE II

Computation step	Initial state	Transition state	Input	Carry	Output	Result
1	1	9	6+3	0	9	9
2	9	2	0+2	0	2	26

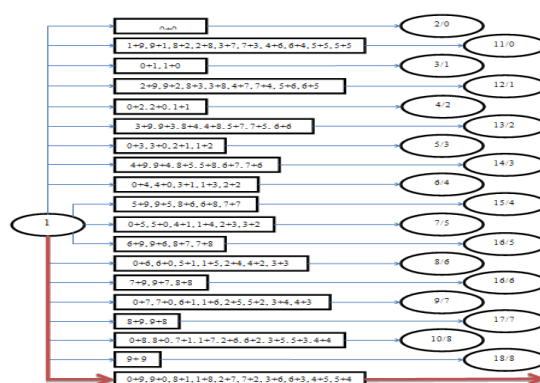


FIGURE III DECIMAL AUTOMATON ADDER PART I

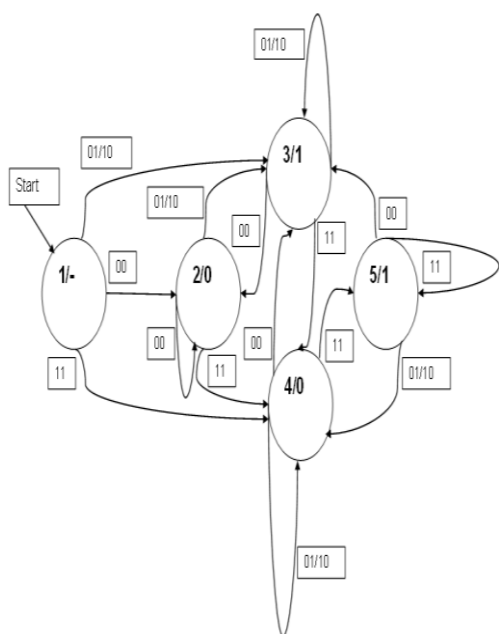


FIGURE IV FINITE BINARY BOOLEAN AUTOMATON ADDER

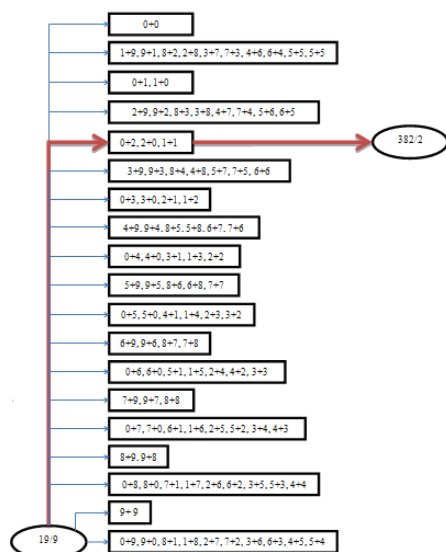


FIGURE V DECIMAL AUTOMATON ADDER PART2

VII. CONCLUSION

It was found in the literature that there are researchers who claim that quantum computers could offer a computational speed up for some computational tasks. The present study has investigated the difference between classical computing and quantum computing for the computational task addition. In the

first instance, both circuit diagrams for the half adder based on the literature were compared for each paradigm. It was established that they are equal in performance as they perform the same operation through different logic gates. Furthermore, it was found for both paradigms that it is possible to evaluate the computational performance through a finite state adder automaton model of computation. This implied that they were equal in performance, and can be categorised as progressing in linear time according to computational complexity theory.

In conclusion, this study suggested that further research into how quantum mechanics can help to improve performance of computing the computational task addition. However, it is believed that to perform addition more effectively a different approach is required. Some literature is indicated that this can be achieved through multi-value logic. As it is noticed that both the classical and quantum circuits have different types of fundamental units, these are 'Bits' and 'Qubits' respectively. Nevertheless they both operate on a binary number system. This raises the question of whether computation through quantum mechanics can be achieved that enables the usage of a higher radix to compute addition faster than binary based computation.

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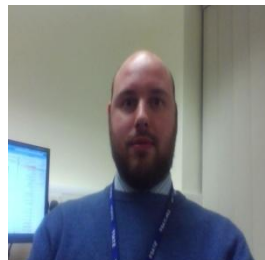
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