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# WAVE PROPAGATION IN A ROTATING FIBRE-REINFORCED PORO-ELASTIC SOLID UNDER THE ACTION OF UNIFORM MAGNETIC FIELD

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**Abstract :** In this article plane wave propagation in a rotating fibre-reinforced medium under the action of a magnetic field of constant magnitude has been investigated. The material is supposed to be porous in nature and contains voids. Following the concept of Cowin and Nunziato [1], the governing equations of motion have been written in tensor notation taking into account of rotation, magnetic field effect and presence of voids in the medium and the possibility of plane wave propagation has been examined. Some particular cases have been derived from our general results. Effects of various parameters on the velocity of wave propagation have been presented graphically.

**Keywords :** Fibre-reinforced media, media with voids, magnetic permeability, electric conductivity, plane wave.

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## 1. Introduction

Fibre-reinforced materials are in use through ages for reinforcing solids as per need. In ancient times people used horsehair in mortar and straws in mud to generate more strength in solids without adding much weight to them. In the early 1900's asbestos fibres were in use in concrete and later in 1950's replacement of such fibres were needed due to observed health hazard and subsequently steel, glass, polypropylene like synthetic fibres were introduced in concrete. The process of fibre-reinforcing continues till date with advanced technology. The process of fibre-reinforcing is developing and in use in different fields. A fibre-reinforced composite (FRC) is a composite building material that consists of three components: (i) the fibres as the discontinuous or dispersed phase, (ii) the matrix as the continuous phase, and (iii) the fine interphase region, also known as the interface. This is a type of advanced composite group, which makes use of rice husk, rice hull, and plastic as ingredients. Carbon fibre is ideal as a strengthening member in umbilicals and flexible pipes for deep water installations. Most concrete construction includes steel reinforcing, at least nominally. Effects of earthquakes on artificial structures are of prime importance to engineers and architects. During an earthquake and similar disturbances a structure is excited into a more or less violent vibration, with resulting oscillatory stresses, which depend both upon the ground vibration and physical properties of the structure [Richter[2]]. Considering this phenomenon wave propagation in a reinforced medium plays a very important role in civil engineering and geophysics.

In classical theory of elasticity the governing equations are based on the generalized Hooke's law indicating stress components as linear functions of the strain components. In developing the governing equations one intuitive assumption that a solid is a continuum, played an important role. There is no denying the fact that although this assumption is valid for a wide class of solids, there remains a lot where this assumption seems to be inadequate. Geological materials like rocks and soils, and manufactured materials like ceramics and pressed powder and many others belong to this class where material voids play quite a significant role. To study the effects of loadings on such materials Cowin and Nunziato [1] developed a new theory in which they have introduced a new parameter $\phi$ in the stress-strain relations. The parameter $\phi$ represents the change in local volume fraction of the solid with respect to a reference frame. The change in volume fraction is taken as an independent kinematic variable. Some basic theorems related to materials with voids are discussed by Cowin [3], Goodman and Cowin [4], Cowin and Nunziato [5], Puri [6], Chandrasekharaiah [7] and Issan [8]. They developed a linear theory applicable to elastic materials with voids for the mathematical study of the mechanical behavior of such materials. This linearized theory of elastic materials with voids is a generalization of the classical theory of elasticity and reduces to it when the dependence on change in volume fraction and its gradient are suppressed. The modified linear theory when applied to the propagation of longitudinal waves in a porous medium shows some distinct characters of its own. The propagation of longitudinal waves in an elastic medium is seen to be significantly affected due to the presence of voids in the medium while the transverse wave propagation remains unaffected. Increasing uses of these materials suggest that the study of solid mechanics problems needs to be extended to fibre-reinforced media as well as medium with voids.

The studies of propagation, reflection and transmission of waves are of a great interest to seismologists. Such studies help them to obtain knowledge about the rock structures as well as their elastic properties and at the same time information regarding minerals and fluids present inside the earth. The concept of continuous self-reinforcement at every point of an elastic solid is due to Belfied et al. [9]. Afterwards some works are observed to be done by Verma and Rana [10], Sengupta and Nath [11], Hashin and Rosen [12], Singh and Singh [13], Pradhan et al. [14], Chattopadhyay et al. [15], Singh [16, 17] etc.

It is quite evident that propagation of elastic waves in solids depends upon the material characteristics of the solid. Presence of voids in the solid or reinforcement in the medium is expected to affect the nature of propagation. A number of problems related to wave propagation in elastic media with voids have been attempted by various investigators. Among them mention may be made of Chandersekharaiah [18], Tomar and Singh [19, 20], Wright [21], Dey et al. [22, 23], Maity et al. [24, 25]. A number of discussions relating wave propagation in rotating isotropic or transversely isotropic media were reported in literature, some of which are the works of Sharma ans Kaur [26], Clarke and Burdness [27], Ailawalia and Budhiraja [28], Othman et al.[29], Gupta and Gupta [30] etc.

The present discussion aims at the study of the propagation of plane waves in a rotating fibre-reinforced medium with voids. A magnetic field of uniform magnitude is supposed to be acting on the medium but there is no body force. The governing equations of motion are framed taking into account of the reinforced vector, void characteristics of the material, rotational effects and the applied magnetic field. Equations have been presented using tensor notations. Possibilities of plane wave propagation in the medium have been studied in this discussion. Effects of reinforced parameter, rotation, applied magnetic field, and void character of the material on plane wave propagation have been examined. Some particular cases have been derived from our general discussion. Finally, some graphical presentations have been made to assess the effects of various parameters on the possible wave velocity of the plane wave propagation in fibre-reinforced media.

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## 2. Field equations

Following Belfield et al. [9] the stress-strain relations for linearly fibre-reinforced elastic medium may be expressed in tensor form as

$$\tau_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu_T\varepsilon_{ij} + \alpha^*(a_k a_m \varepsilon_{km}\delta_{ij} + a_i a_j \varepsilon_{kk}) + 2(\mu_L - \mu_T)(a_i a_k \varepsilon_{kj} + a_j a_k \varepsilon_{ki}) + \beta^*(a_k a_m a_i a_j \varepsilon_{km}) + \beta\phi\delta_{ij} \quad (1)$$

where $\tau_{ij}$ are the cartesian components of the stress tensor; $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the strain components, related to the displacement vector $u_i$; $\lambda, \mu_T$ are elastic constants; $\alpha^*, \beta^*, (\mu_L - \mu_T)$ are reinforcement parameters; $\beta$ is a void parameter and $\mathbf{a} = (a_1, a_2, a_3)$, represents the direction of reinforcement.

For a rotating elastic medium the equation of motion, in absence of body force, can be written as

$$\tau_{ij,j} = \rho[\ddot{u}_i + \{\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\mathbf{\Omega}\times\mathbf{u}\}_i] \cdots\cdots(2)$$

In (2), $\rho$ denotes the material density, $\mathbf{\Omega}$ is the angular velocity vector, overhead dot represents differentiation with respect to time and the suffix $i$ after second bracket represent the $i$th component of the vector inside.

If, in addition, the solid is under the action of magnetic field $\mathbf{H}$, then the governing field equations involving the displacement $\mathbf{u} = u_i(x,t)$ and the volume fraction $v(\mathbf{x},t)$, for a homogeneous elastic material containing a distribution of void pores, in absence of body force, may be written as

$$(\lambda + \mu_T)u_{k,ki} + \mu_T u_{i,kk} + \alpha^*(a_k a_m u_{k,mi} + a_i a_j u_{k,kj}) + (\mu_L - \mu_T)[a_i a_k u_{k,jj} + a_i a_k u_{j,kj} + a_j a_k (u_{k,ij} + u_{i,kj})] +$$

$$\beta^* a_i a_j a_k a_m u_{k,mj} + (\mathbf{J}\times\mathbf{B})_i = \rho[\ddot{u}_i + \{\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\mathbf{\Omega}\times\mathbf{u}\}_i] \cdots\cdots(3)$$

and

$$\alpha\varphi_{,kk} - \omega_d\dot{\varphi} - \xi\varphi - \beta u_{k,k} = \rho\bar{k}\ddot{\varphi} \cdots\cdots(4)$$

The term $\mathbf{J}\times\mathbf{B}$ in (3) arises from the presence of the applied magnetic field . Due to the application of the initially applied magnetic field $\mathbf{H}_0$, an induced magnetic field $\mathbf{h}$, an induced electric field $\mathbf{E}$ and a current density $\mathbf{J}$ are developed. For a slowly moving homogeneous electrically conducting medium, the simplified system of linear equations of electrodynamics are

$$\nabla\times\mathbf{h} = \mathbf{J} + \varepsilon_0\mathbf{E}$$
$$\nabla\times\mathbf{E} = -\mu_0\mathbf{h}$$
$$\nabla\cdot\mathbf{h} = 0 \cdots\cdots\cdots(5)$$
$$\mathbf{E} = -\mathbf{u}\times\mathbf{B}$$

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where $\varepsilon_0$ is the electrical conductivity and $\mu_0$ is the magnetic permeability so that $\mathbf{B}=\mu_0\mathbf{H}$ is the magnetic field in the medium due to total magnetic field $\mathbf{H}=\mathbf{H}_0+\mathbf{h}$, arising from applied field $\mathbf{H}_0$ and induced field $\mathbf{h}$.

If we assume that $\mathbf{H}_0=(H_{01},H_{02},H_{03})$ and $\mathbf{\Omega}=(\Omega_1,\Omega_2,\Omega_3)$, then utilizing relations (5) and neglecting products of $\mathbf{h}$ with $\mathbf{u}$ and its derivatives, the governing equations of motion (3) and (4) for an elastic medium with voids under the action of applied magnetic field and rotation may be written in tensor notation as

$$(\lambda+\mu_T)u_{k,ki}+\mu_T u_{i,kk}+\alpha^*(a_k a_m u_{k,mi}+a_i a_j u_{k,kj})+(\mu_L-\mu_T)[a_i a_k u_{k,jj}+a_i a_k u_{j,kj}+a_j a_k(u_{k,ij}+u_{i,kj})]+$$

$$\beta^* a_i a_j a_k a_m u_{k,mj}+\mu_0 H_0^2 u_{j,ji}-\mu_0 H_{0k}H_{0i}u_{j,jk}-\mu_0 H_{0m}H_{0k}u_{k,im}+\mu_0 H_{0m}H_{0k}u_{i,km}-\mu_0^2\varepsilon_0 H_0^2\ddot{u}_i+\mu_0^2\varepsilon_0 H_{0k}H_{0i}\ddot{u}_k+\beta\phi_{,i}$$

$$=\rho(\ddot{u}_i+\Omega_k\Omega_i u_k-\Omega^2 u_i+2\varepsilon_{ijk}\Omega_j\dot{u}_k)\cdots\cdots\cdots(6)$$

$$\alpha\varphi_{,kk}-\omega_d\dot{\varphi}-\xi\varphi-\beta u_{k,k}=\rho\bar{k}\ddot{\varphi}\cdots\cdots\cdots(7)$$

where $\alpha,\xi,\omega_d,\bar{k}$ are nonnegative material constants. In (6), $\varepsilon_{ijk}$ represents the Levi-civita tensor which has a non-zero value only if $i,j,k$ are all distinct and has a value 1 if $i,j,k$ are in cyclic order, whereas, it has a value -1 when they are acyclic.

## 3. Plane wave propagation:

In order to examine the possibility of a plane wave propagation in the medium under consideration we shall assume a solution of governing equations (6) and (7) in the form

$$(u_i,\varphi)=(Ap_i,B)exp\{\tau(qn_s x_s-\omega t)\},i=1,2,3;\tau=\sqrt{-1}\cdots\cdots(8)$$

The speed of the wave is

$$c_n=\frac{\omega}{Re(q)}\cdots\cdots(9)$$

The direction of plane wave propagation is represented by the unit vector $\mathbf{n}=(n_1,n_2,n_3)$, while the direction of particle displacement is denoted by the unit vector $\mathbf{p}=(p_1,p_2,p_3)$. $A$ and $B$ appearing in (8) are constants.

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Substituting (8) into (6) and (7), using $\tau^2=-1, n_k n_k=1$ and setting $\frac{B}{A}=D$, we get

$$(\lambda+\mu_T)p_k q^2 n_i n_k+\mu_T p_i q^2+\alpha^*(a_k a_m n_i n_m+a_i a_j n_k n_j)p_k q^2+(\mu_L-\mu_T)[a_i a_k p_k q^2+a_i a_l p_k q^2 n_k n_l+$$

$$a_j a_k(n_i n_j p_k q^2+n_k n_j p_i q^2)]+\beta^* a_i a_j a_k a_m p_k q^2 n_j+$$

$$\mu_0 H_0^2 p_j q^2 n_j n_i-\mu_0 H_{0k}H_{0i}p_j q^2 n_j n_k-\mu_0 H_{0m}H_{0k}p_k q^2 n_i n_m+\mu_0 H_{0m}H_{0k}p_i q^2 n_k n_m-\mu_0^2\varepsilon_0 H_0^2 p_i\omega^2$$

$$+\mu_0^2\varepsilon_0 H_{0k}H_{0i}p_k\omega^2-\beta D\tau q n_i=\rho[p_i\omega^2-\Omega_k\Omega_i p_k+\Omega^2 p_i+2\varepsilon_{ijk}\Omega_j p_k\tau\omega], i=1,2,3\cdots\cdots(10)$$

$$\alpha q^2+\xi-\omega_d\tau\omega+\beta D^{-1}p_k\tau q n_k=\rho\bar{k}\omega^2\cdots\cdots\cdots(11)$$

Eliminating $D$ from (10) and(11) we get

$$(\lambda+\mu_T)p_k q^2 n_i n_k+\mu_T p_i q^2+\alpha^*(a_k a_m n_i n_m+a_i a_j n_k n_j)p_k q^2+(\mu_L-\mu_T)[a_i a_k p_k q^2+a_i a_l p_k q^2 n_k n_l+$$

$$a_j a_k(n_i n_j p_k q^2+n_k n_j p_i q^2)]+\beta^* a_i a_j a_k a_m p_k q^2 n_j+$$

$$\mu_0 H_0^2 p_j q^2 n_j n_i-\mu_0 H_{0k}H_{0i}p_j q^2 n_j n_k-\mu_0 H_{0m}H_{0k}p_k q^2 n_i n_m+\mu_0 H_{0m}H_{0k}p_i q^2 n_k n_m-\mu_0^2\varepsilon_0 H_0^2 p_i\omega^2$$

$$+\mu_0^2\varepsilon_0 H_{0k}H_{0i}p_k\omega^2-\frac{\beta^2 q^2 p_k n_k n_i}{\rho\bar{k}\omega^2-(\alpha q^2+\xi)+\omega_d\tau\omega}=\rho[p_i\omega^2-\Omega_k\Omega_i p_k+\Omega^2 p_i+2\varepsilon_{ijk}\Omega_j p_k\omega^2], i=1,2,3\cdots(12)$$

Equation (12) is a set of three linear homogeneous equations in $p_k$ of the form

$$p_k\eta_{ik}-p_i\theta=0\cdots\cdots\cdots(13)$$

where

$$\eta_{ik}=q^2[F_{ik}+M_{ik}]+R_{ik}+V_{ik}\cdots\cdots\cdots(14)$$

$$F_{ik}=(\lambda+\mu_T)n_i n_k+\alpha^*(a_k a_m n_i n_m+a_i a_j n_k n_j)+(\mu_L-\mu_T)(a_i a_k+a_i a_l n_k n_l+a_j a_k n_i n_j)+\beta^* a_i a_j a_k a_m n_m n_j$$

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$$\theta = -[\mu_T + (\mu_L - \mu_T)a_j a_k n_k n_j] + \mu_0^2 \varepsilon_0 \omega^2 H_0^2 - q^2 \mu_0 H_{0m} H_{0k} n_m n_k + \rho(\Omega^2 + \omega^2)$$

$$M_{ik} = \mu_0 H_0^2 n_i n_k - \mu_0 H_{0i} H_{0j} n_j n_k - \mu_0 H_{0m} H_{0k} n_i n_m + \frac{\mu_0^2 \varepsilon_0 \omega^2 H_{0k} H_{0i}}{q^2}$$

$$R_{ik} = \rho \Omega_k \Omega_i - 2\rho \varepsilon_{ijk} \omega_j \tau \omega \cdots\cdots\cdots (15)$$

$$V_{ik} = \frac{\beta^2 q^2 n_i n_k}{\rho \bar{k} \omega^2 - (\alpha q^2 + \xi) + \omega_d \tau \omega}$$

Rewriting (13) as

$$p_k (\eta_{ik} - \theta \delta_{ik}) = 0 \cdots\cdots\cdots (16)$$

and noting that not all $p_k$'s are zero, it follows that

$$|\eta_{ik} - \theta \delta_{ik}| = 0 \cdots\cdots\cdots (17)$$

The determinantal equation (17) yields an algebraic equation in $q^2$ with complex coefficients which will determine the wave speed $c_n$ in (9). It is clear that the velocity of the plane wave propagation depends on the terms $F_{ik}$ arising from the elastic behavior of the fibre-reinforced material and the direction $n_i$ of propagation of the wave, the terms $M_{ik}$ arising from the applied magnetic field, the terms $R_{ik}$ arising from the rotation of the medium and the terms $V_{ik}$ arising from the void character of the material.

As a particular derivation from our general results above we consider a fibre-reinforced elastic solid with voids, the reinforcement being done in the direction of unit vector $\mathbf{a} = (a_1, a_2, a_3)$. The solid in rotating with uniform angular velocity $\mathbf{\Omega} = \Omega(0,0,1)$ and is under uniform magnetic field $\mathbf{H}_0 = H_0(0,1,0)$. We like to investigate propagation of a plane wave in the medium in a direction specified by the unit vector $\mathbf{n} = (0, n_2, n_3)$.

In this case we rewrite equation (16) in the form

$$p_k D_{ik} = 0, i = 1,2,3 \cdots\cdots\cdots (18)$$

where $D_{ik} = \eta_{ik} - \theta \delta_{ik}$.

Writing $x = \dfrac{q}{\omega}$,we find that,in this particular case

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$$D_{11}=a_{11}x^2-R,\ D_{12}=a_{12}x^2+2\rho\Gamma\tau,\ D_{13}=D_{31}=a_{13}x^2,\ D_{21}=a_{12}x^2-2\rho\Gamma\tau,\ D_{22}=x^2[a_{22}+n_2^2\gamma(x^2)]-R,$$

$$D_{23}=D_{32}=x^2[a_{23}+n_2n_3\gamma(x^2)],\ D_{33}=x^2[a_{33}+n_3^2\gamma(x^2)]-R_1$$

where

$$a_{11}=(\mu_L-\mu_T)a_1^2+\beta^*a_1^2(a_2n_2+a_3n_3)^2+\mu_T,$$

$$a_{12}=(\alpha^*+\mu_L-\mu_T)a_1n_2(a_2n_2+a_3n_3)+(\mu_L-\mu_T)a_1a_2+\beta^*a_1a_2(a_2n_2+a_3n_3)^2,$$

$$a_{13}=(\alpha^*+\mu_L-\mu_T)a_1n_3(a_2n_2+a_3n_3)+(\mu_L-\mu_T)a_1a_3+\beta^*a_1a_3(a_2n_2+a_3n_3)^2,$$

$$a_{22}=(\lambda+\mu_T)n_2^2+\{2\alpha^*+3(\mu_L-\mu_T)\}a_2n_2(a_2n_2+a_3n_3)+(\mu_L-\mu_T)a_2^2+\beta^*a_2^2(a_2n_2+a_3n_3)^2+\mu_T,$$

$$a_{23}=(\lambda+\mu_T)n_2n_3+(\alpha^*+\mu_L-\mu_T)(a_2n_2+a_3n_3)(a_2n_3+a_3n_2)+(\mu_L-\mu_T)a_2a_3+\beta^*a_2a_3(a_2n_2+a_3n_3)^2$$

$$a_{33}=(\lambda+\mu_T)n_3^2+\{2\alpha^*+3(\mu_L-\mu_T)\}a_3n_3(a_2n_2+a_3n_3)+(\mu_L-\mu_T)a_3^2+\beta^*a_3^2(a_2n_2+a_3n_3)^2+\mu_T+\mu_0H_0^2n_3^2$$

$$R=\varepsilon_0\mu_0^2H_0^2+\rho(1+\Gamma^2),\ R_1=\varepsilon_0\mu_0^2H_0^2+\rho,\ \gamma(x^2)=\frac{\beta^2}{\rho\bar{k}\omega^2-\xi+\omega_d\tau\omega-\alpha\omega^2x^2}\quad and\quad \Gamma=\frac{\Omega}{\omega}.$$

The determinantal equation $|D_{ik}|=0$ gives following two algebraic equations of degree 5 and 2 in $x^2$;

$$\chi_1x^{10}+\chi_2x^8+\chi_3x^6+\chi_4x^4+\chi_5x^2+\chi_6=0\cdots\cdots\cdots(19)$$

and

$$\chi_7x^4+\chi_8x^2+\chi_9=0\cdots\cdots\cdots(20)$$

where

$$\chi_1=\mu_TA_2(a_{22}a_{33}-a_{23}^2),$$

$$\chi_2=-\mu_TA_2(a_{22}R_1+a_{33}R)+(a_{22}a_{33}-a_{23}^2)(\mu_TB_2-RA_2)+\mu_TA_1(a_{33}n_2^2+a_{22}n_3^2-2a_{23}n_2n_3),$$

$$\chi_3=\mu_TA_2RR_1-(a_{22}R_1+a_{33}R)(\mu_TB_2-RA_2)+(a_{22}a_{33}-a_{23}^2)(\mu_TC_2-RB_2)-\mu_TA_1(R_1n_2^2+Rn_3^2)$$

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$$+(\mu_T B_1 - RA_1)(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3) - 4a_{33}A_2\rho^2\Gamma^2,$$

$$\chi_4 = \mu_T B_2 RR_1 - (a_{22}R_1 + a_{33}R)(\mu_T C_2 - RB_2) - RC_2(a_{22}a_{33} - a_{23}^2) - A_2R^2R_1 - (\mu_T B_1 - RA_1)(R_1n_2^2 + Rn_3^2)$$

$$-RB_1(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3) + 4\rho^2\Gamma^2(R_1A_2 - n_3^2A_1 - a_{33}B_2),$$

$$\chi_5 = \mu_T C_2 RR_1 - B_2R^2R_1 + RC_2(a_{22}R_1 + a_{33}R) + RB_1(R_1n_2^2 + Rn_3^2) + 4\rho^2\Gamma^2(R_1B_2 - n_3^2B_1 - a_{33}C_2),$$

$$\chi_6 = R_1C_2(4\rho^2\Gamma^2 - R^2),\ \ \chi_7 = \mu_T(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3),$$

$$\chi_8 = R(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3) + \mu_T(R_1n_2^2 + Rn_3^2),\ \ \chi_9 = R(R_1n_2^2 + Rn_3^2) - 4\rho^2\Gamma^2 n_3^2,$$

$$A_1 = -\alpha\omega^2\beta^2,\ A_2 = \alpha^2\omega^4,\ B_1 = (\rho\bar{k}\omega^2 - \xi)\beta^2,\ B_2 = -2\alpha\omega^2(\rho\bar{k}\omega^2 - \xi),\ C_2 = (\rho\bar{k}\omega^2 - \xi)^2 + (\omega_d\omega)^2.$$

A real and positive root of equations $(19)$ and $(20)$ will indicate a plane wave propagation in the medium. The speed of the wave will be $\dfrac{1}{|x|}$.

Furthermore, if the wave is propagating in the direction $\mathbf{n} = (0,0,1)$ and $\mathbf{a} = (0,0,a_3)$, then the determinantal equation $(17)$ becomes

$$\begin{vmatrix} D_{11}^{'} & 2\tau\rho\Gamma & 0 \\ -2\tau\rho\Gamma & D_{22}^{'} & 0 \\ 0 & 0 & D_{33}^{'} \end{vmatrix} = 0 \cdots\cdots\cdots (21)$$

where

$$D_{11}^{'} = D_{22}^{'} = x^2\mu_T - \varepsilon_0\mu_0^2H_0^2 - \rho(1+\Gamma^2)$$

$$D_{33}^{'} = x^2[(\lambda + 2\mu_T) + 2\alpha^* a_3^2 + 4(\mu_L - \mu_T)a_3^2 + \beta^* a_3^4 + \mu_0 H_0^2 + \gamma(x^2)] - \varepsilon_0\mu_0^2H_0^2 - \rho.$$

The determinantal equation $(21)$ indicates plane waves of different characters and velocities propagating in the medium:

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(i) plane wave propagating with speed

$$V_1 = \sqrt{\frac{\Upsilon - \sqrt{\Upsilon^2 - 4A_5A_6R_1\alpha\omega^2}}{2A_6R_1}} \cdots\cdots\cdots(22)$$

depending upon the reinforcement parameters, applied magnetic field, void parameters of the medium, where

$$\Upsilon = (R_1\alpha\omega^2 + A_5A_6 + \beta^2) \ \ and \ \ A_5 = (\lambda + 2\mu_T) + 2\alpha^* a_3^2 + 4(\mu_L - \mu_T)a_3^2 + \beta^* a_3^4 + \mu_0 H_0^2, \ \ A_6 = \rho\bar{k}\omega^2 - \xi.$$

The velocity $V_1$ is observed to be basically dependent on void parameters. If the medium has no voids, the velocity $V_1$ does not exist, even if the magnetic field or reinforcement is there. Again, if the medium has voids but no magnetic field applied, then there is a plane wave propagating in the reinfoced medium with velocity

$$V_1' = \sqrt{\frac{\Upsilon_1 - \sqrt{\Upsilon_1^2 - 4A_5'A_6\rho\alpha\omega^2}}{2A_6\rho}}$$

depending upon the reinforcement parameters, applied magnetic field, void parameters of the medium, where

$$\Upsilon_1 = (\rho\alpha\omega^2 + A_5'A_6 + \beta^2) \ \ and \ \ A_5' = (\lambda + 2\mu_T) + 2\alpha^* a_3^2 + 4(\mu_L - \mu_T)a_3^2 + \beta^* a_3^4.$$

(ii)plane wave propagating with speed

$$V_2 = \sqrt{\frac{A_5}{R_1}} \cdots\cdots\cdots(23)$$

depending upon the direction of reinforcement, applied magnetic field, void parameters and rotation parameter of the medium.

(iii) plane wave propagating with speed

$$V_3, V_4 = \sqrt{\frac{\mu_T}{\varepsilon_0\mu_0^2 H_0^2 + \rho(1 \pm \Gamma)^2}} \cdots\cdots\cdots(24)$$

($V_3$ for $+$ sign and $V_4$ for $-$ sign) depending upon the applied magnetic field, rotation of the medium but not the direction of reinforcement and void parameters .

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## 3. Numerical results and discussions:

The present study focuses on the effects of fibre reinforcement, rotation, magnetic field and void pores of the medium on the propagation of plane wave in a solid. To observe the effects numerically we have adopted three sets of values of relevant parameters from the works of Othman et al. [31], Markham[32], Zorammuana[33] as given below;

$$\lambda = 9.4\times10^{9}\,N\cdot m^{-2},\ \mu_T = 1.89\times10^{9}\,N\cdot m^{-2},\ \mu_L = 2.45\times10^{9}\,N\cdot m^{-2},\ \rho = 1.7\times10^{3}\,Kg\cdot m^{-3},$$

$$\lambda = 5.65\times10^{9}\,N\cdot m^{-2},\ \mu_T = 2.46\times10^{9}\,N\cdot m^{-2},\ \mu_L = 5.66\times10^{9}\,N\cdot m^{-2},\ \rho = 2.26\times10^{3}\,Kg\cdot m^{-3},$$

$$\lambda = 7.59\times10^{9}\,N\cdot m^{-2},\ \mu_T = 3.5\times10^{9}\,N\cdot m^{-2},\ \mu_L = 7.07\times10^{9}\,N\cdot m^{-2},\ \rho = 1.6\times10^{3}\,Kg\cdot m^{-3},$$

$$\alpha = 3.668\times10^{-4}\,N,\ \beta = 1.13849\times10^{11}\,N\cdot m^{-2},\ \xi = 1.475\times10^{12}\,N\cdot m^{-2},\ \bar{k} = 1.753\times10^{-15}\,N\cdot m^{-2},$$

$$\alpha^{*} = -1.28\times10^{9}\,N\cdot m^{-2},\ \beta^{*} = .32\times10^{9}\,N\cdot m^{-2}.$$

Adopting these values in our numerical computation, we have tried to analyze the trends of the behaviour of plane wave propagating in the medium under different media condition and load condition. Figs. 1 and 2 display the effect of fibre-reinforced parameters on possible real wave velocities $V_1$ and $V_2$ obtained from equation (21), with magnetic field $H_0$ when wave propagates along $x_3$ axis. It is observed that in both the cases when magnetic field is absent then wave velocity is maximum for different values of fibre-reinforced parameter and tends to zero when magnetic field increases. It is also found that magnitudes of wave velocity increases with the increased values of $(\mu_L - \mu_T)$. The variation of two possible real wave velocities $V_3$ and $V_4$ with applied magnetic field $H_0$ for various values of $\Gamma$ are shown in Figures 3 and 4 when wave propagates along $x_3$ axis. In Fig. 3 magnitude of wave velocity $V_3$ is greater when rotation is absent and magnitude of $V_3$ is smaller when $\Gamma$ increases but reverse behaviours are reflected in Fig. 4. It is to be noted that in both Figs. 3 and 4 maximum values of wave velocities $V_3$ and $V_4$ occur when applied magnetic field is zero for all the values of rotation parameter $\Gamma$. Influence of reinforcement parameter $\mu_T$ on wave velocities $V_3$ and $V_4$ are shown in Figs. 5 and 6. It is obvious from the Figs. that in both the cases magnitudes of wave velocity increases with the increased values of $\mu_T$.

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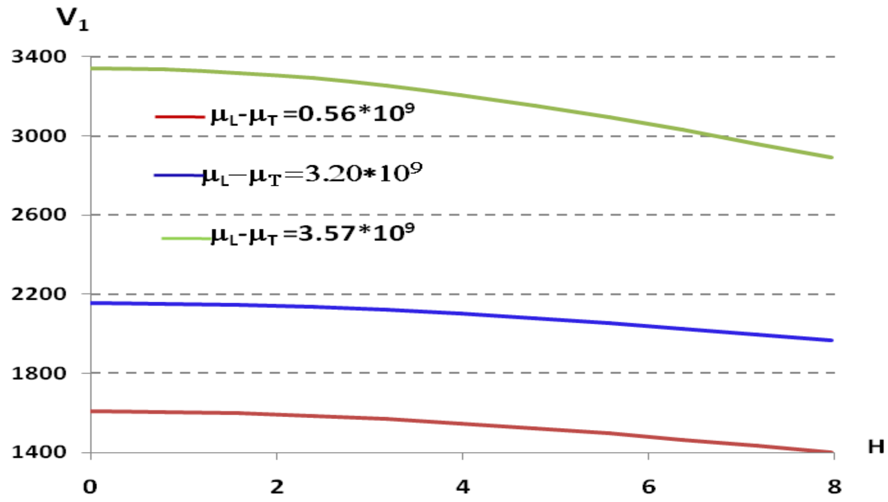


Fig. 1 Effect of reinforcement parameter on wave velocity $V_1$

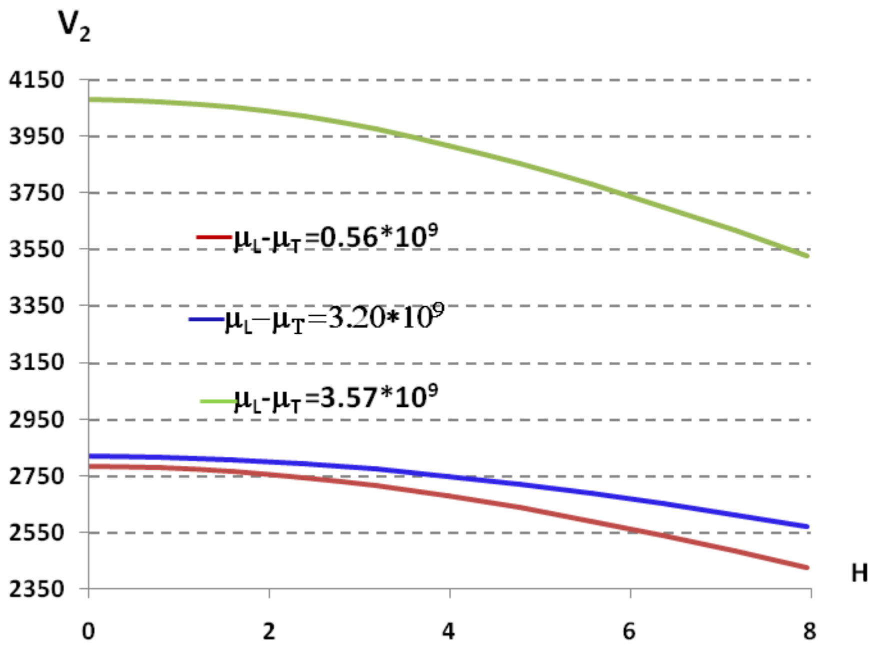


Fig. 2 Effect of reinforcement parameter on wave velocity $V_2$

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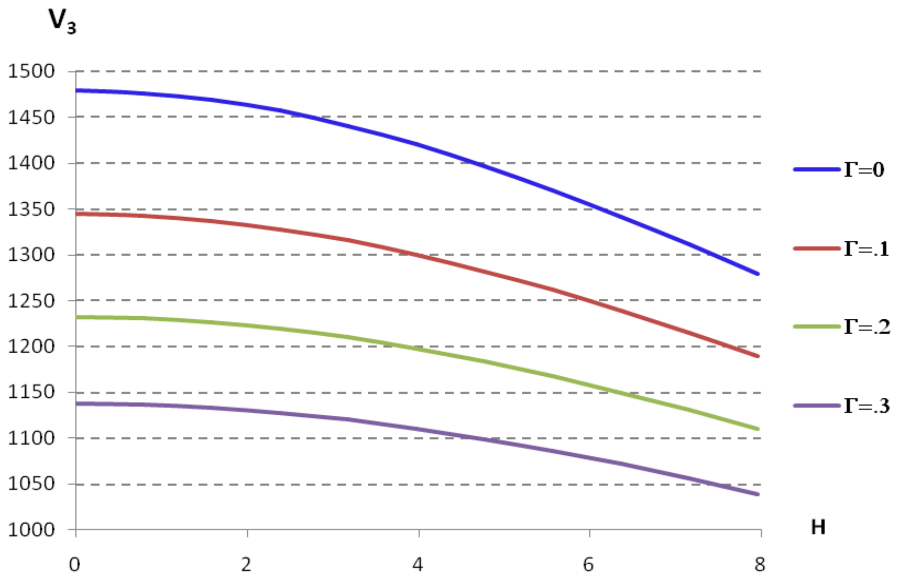


Fig. 3 Effect of rotation parameter $\Gamma$ on wave velocity $V_3$

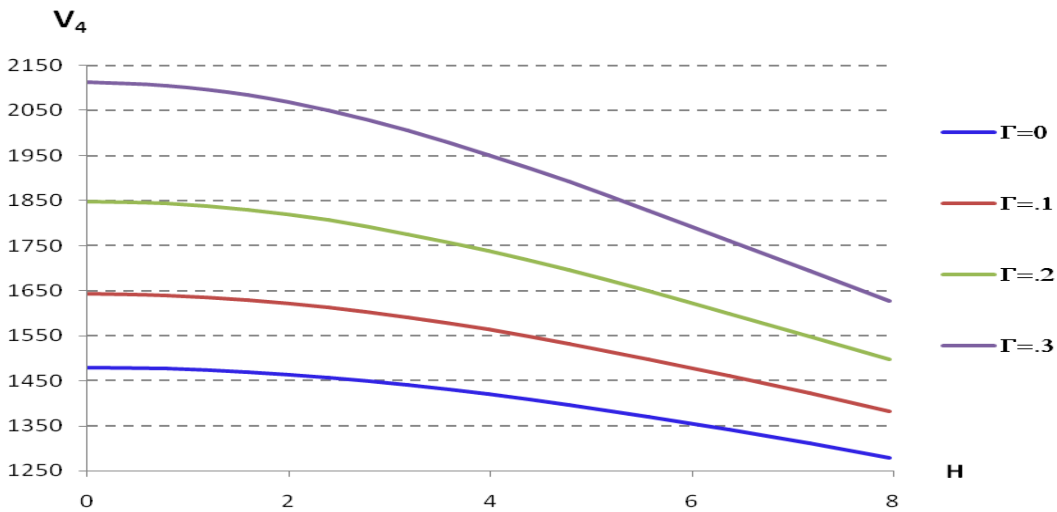


Fig. 4 Effect of rotation parameter $\Gamma$ on wave velocity $V_4$

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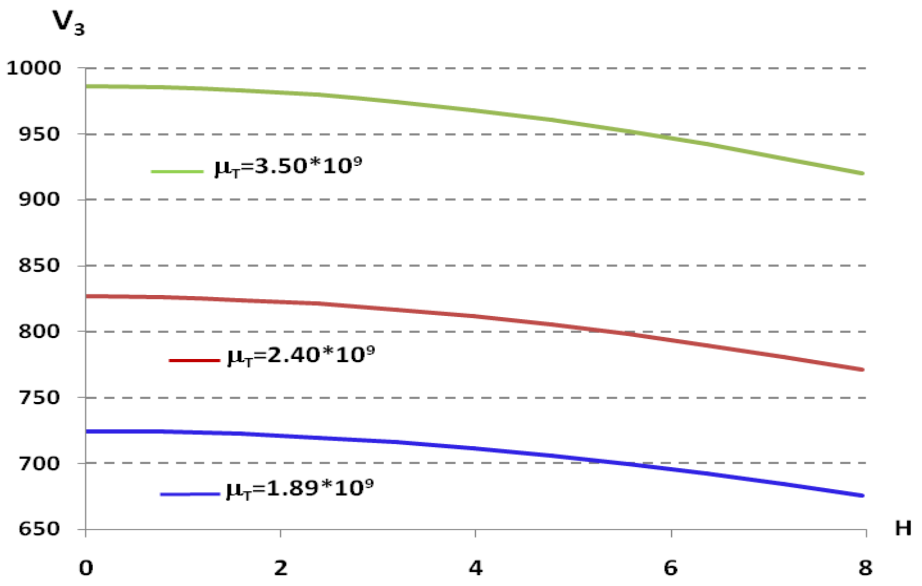


Fig. 5 Effect of reinforcement parameter $\mu_T$ on wave velocity $V_3$

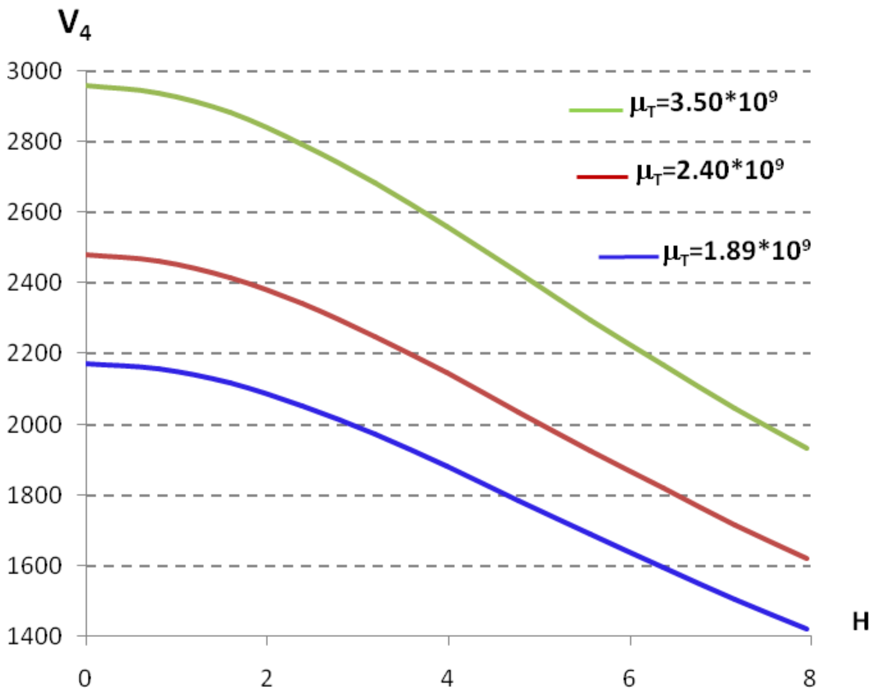


Fig. 6 Effect of reinforcement parameter $\mu_T$ on wave velocity $V_4$

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