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# A note on annihilating ideal graph of $Z_n$

**N. Mohamed Rilwan**[1] **and E. Chidambaram**[2]
[1,2]Department of Mathematics
Sadakathullah Appa College,
Tirunelveli - 627 011
Email: rilwan2020@gmail.com, chidambaram.e@gmail.com

**Abstract**

Let $R$ be a commutative ring with identity and $\mathbb{A}^*(R)$ the set of non-zero ideals with non-zero annihilators. The *annihilating-ideal graph* of $R$ is defined as the graph $\mathbb{AG}(R)$ with the vertex set $\mathbb{A}^*(R)$ and two distinct vertices $I_1$ and $I_2$ are adjacent if and only if $I_1I_2 = (0)$. In this paper, we obtain a characterization for the annihilating-ideal graph $\mathbb{AG}(R)$ to be unicyclic, claw-free and outerplanar when $R = \mathbb{Z}_n$.

**Keywords:** claw-free graph, outer planar graph, unicyclic graph.
**Subject Classification:** 05C38, 05C75, 13A15.

## 1 Introduction

The study of algebraic structures using the properties of graphs became an exciting research topic in the past years leading to many fascinating results and questions. There are many papers assigning graphs to rings, groups and semigroups. Let $R$ be a commutative ring with identity. In [1], D. F. Anderson and P. S. Livingston associate a graph called *zero-divisor graph*, $\Gamma(R)$ to $R$ with vertices $Z(R)^*$, the set of non-zero zero-divisors of $R$ and for two distinct $x, y \in Z(R)^*$, the vertices $x$ and $y$ are adjacent if and only if $xy = 0$ in $R$. Recently M. Behboodi and Z. Rakeei [4, 5] have introduced and investigated the annihilating-ideal graph of a commutative ring. We call an ideal $I_1$ of $R$, an *annihilating-ideal* if there exists a non-zero ideal $I_2$ of $R$ such that $I_1I_2 = (0)$. For a non-domain commutative ring $R$, let $J(R)$ be the Jacobson radical of $R$ and $\langle x \rangle$ be the ideal of $R$ generated by $x$ and $\mathbb{A}^*(R)$ be the set of non-zero ideals with non-zero annihilators. The *annihilating-ideal graph* of $R$ is defined as the graph $\mathbb{AG}(R)$ with the vertex set $\mathbb{A}^*(R)$ and two distinct vertices $I_1$ and $I_2$ are adjacent if and only if $I_1I_2 = (0)$.

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An ideal $I$ of $R$ is called *nil-ideal* if there exists a positive integer $n$ such that $I^n = 0$ and $I^{n-1} \neq (0)$. This integer $n$ is called the nilpotency of the ideal. The *annihilator* of $a \in R$ is the set of all elements $x$ in $R$ such that $ax = 0$ and is denoted by $ann(a)$. Let $I$ be a non-zero ideal in $R$, $ann(I) = \{x \in R : xa = 0 \text{ forall } a \in I\}$. For basic definitions on rings, one may refer [2, 8].

Let $G = (V, E)$ be a simple connected graph. For a vertex $v \in V(G)$, the *neighborhood* (*degree*) of $v$, denoted by $N_G(v)$ ($deg_G(v)$), is the set (number) of vertices other than $v$ which are adjacent to $v$. For basic definitions on graphs, one may refer [6, 11]. In this paper, we obtain a characterization for the annihilating-ideal graph $\mathbb{AG}(R)$ to be unicyclic, claw-free and outerplanar when $R = \mathbb{Z}_n$.

## 2 *Some basic properties of* $\mathbb{AG}(R)$

Note that $\tau(n)$ is the number of all positive divisors of $n$.

**Lemma 2.1** *Let* $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ *with* $p_1, p_2, \ldots, p_k$ *are distinct primes,* $\alpha_1, \alpha_2, \ldots, \alpha_k$ *are positive integers,* $R = \mathbb{Z}_n$ *where* $n \neq p$ *and* $p$ *is a prime. Then the following are true in* $\mathbb{AG}(R)$.

i. $|V(\mathbb{AG}(R))| = \tau(n) - 2$;
ii. $|V(\mathbb{AG}(R))| = 1$ if and only if $R = \mathbb{Z}_{p^2}$ where $p$ is a prime;
iii. If $|V(\mathbb{AG}(R))| \geq 2$, then $\mathbb{AG}(R)$ has no isolated vertex.

*Proof*. **Case (i).** We know that the number of ideals in $\mathbb{Z}_n$ is equal to the number of all positive divisors of $n$. Note that $\{0\} \notin V(\mathbb{AG}(R))$ and $ann(R) = \{0\}$. By the definition $A(R), |V(\mathbb{AG}(R))| \leq \tau(n) - 2$. Let $I$ be a non-trivial ideal in $R$. Then $I = \langle d \rangle$ where $d \mid n$ and $d \neq 1, d \neq n$.

**Subcase 1.** $d \neq \frac{n}{d}$.

Note that $d \mid n$ and $\frac{n}{d} \mid n$. Let $J = \langle \frac{n}{d} \rangle$. Since $n = d.\frac{n}{d}, \frac{n}{d} \in ann(I)$. Then $ann(I) \neq \{0\}$ and so $I \in V(\mathbb{AG}(R))$. In this case, $|V(\mathbb{AG}(R))| = \tau(n) - 2$.

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**Sub case 2.** $d = \frac{n}{d}$.

Here $n = d^2$. Then $ann(I) \supseteq I$ and $I \in V(\mathrm{AG}(R))$. Thus, $|V(\mathrm{AG}(R))| = \tau(n) - 2$.

**Case (ii).** Proof is trivial.

**Case (iii).** Let $I \in V(\mathrm{AG}(R))$. Then $I = < d >$ where $d | n$ and $1 \neq d \neq n$.

**Sub case 1.** $d \neq \frac{n}{d}$.

Note that $d | n, \frac{n}{d} | n$ and $\frac{n}{d} \neq 1, \frac{n}{d} \neq n$. Let $J = < \frac{n}{d} >$. Then $J \in V(\mathrm{AG}(R))$. Since $\frac{n}{d} \in ann(I), J \subseteq ann(I)$. Similarly, $I \subseteq ann(J)$. Thus, $IJ = \{0\}$ and so $I$ and $J$ are adjacent in $\mathrm{AG}(R)$.

**Sub case 2.** $d = \frac{n}{d}$.

Note that $n = d^2 = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}, \alpha_i$'s are even and so $\alpha_i \geq 2$ for all $i, 1 \leq i \leq k$. Since $|V(\mathrm{AG}(R))| \geq 2$ and above case (ii), $n \neq p_1^2$. Let $J = < \frac{n}{p_1} >$. Then $J \in V(\mathrm{AG}(R))$ and $I \neq J$. Also, $I \subseteq ann(J)$ and $J \subseteq ann(I)$. Thus, $IJ = \{0\}$ and so $I$ and $J$ are adjacent in $\mathrm{AG}(R)$.

**Lemma 2.2** *Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ with $p_1, p_2, \ldots, p_k$ are distinct primes, $\alpha_1, \alpha_2, \ldots, \alpha_k$ are positive integers, $R = \mathbb{Z}_n$ where $n \neq p$ and $p$ is a prime. Let $|V(\mathrm{AG}(R))| \geq 2$. Then the following are true in $\mathrm{AG}(R)$.*

i. $\mathrm{AG}(R)$ contains a vertex of degree one

ii. $\mathrm{AG}(R)$ is neither Eulerian nor Hamiltonian.

*Proof.* (i). Since $|V(\mathrm{AG}(R))| \geq 2$, $R \neq \mathbb{Z}_{p^2}$ where $p$ is a prime. Let $I = < p_1 >$ and $J = < \frac{n}{p_1} >$ be two distinct vertices in $V(\mathrm{AG}(R))$. Then $I$ is only adjacent to $J$ in $\mathrm{AG}(R)$ and so $\deg_{\mathrm{AG}(R)}(I) = 1$. (ii) Proof follows from above (i).

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In the following theorem, we charactarize when $\mathsf{AG}(R)$ is triangle.

**Lemma 2.3** *Let* $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ *with* $p_1, p_2, \ldots, p_k$ *are distinct primes,* $\alpha_1, \alpha_2, \ldots, \alpha_k$ *are positive integers,* $R = \mathsf{Z}_n$ *where* $n \neq p$ *and* $p$ *is a prime. Then* $\mathsf{AG}(R)$ *contains a triangle if and only if* $n$ *is any one of the following:*

i. $n = p_1^{\alpha_1}$ $(\alpha_1 \geq 5)$
ii. $n = p_1^2 p_2^2$
iii. $n = p_1^{\alpha_1} p_2^{\alpha_2}, \alpha_i \geq 3$ for some $1 \leq i \leq 2$
iv. $k \geq 3$.

*Proof.* (i) Let $S = \{\langle p_1^{\alpha_1 - 1}\rangle, \langle p_1^{\alpha_1 - 2}\rangle, < p_1^{\alpha_1 - 3} >\} \subset V(\mathsf{AG}(R))$. Then $\langle S\rangle = K_3$ in $\mathsf{AG}(R)$.

(ii) Let $S = \{\langle p_1 p_2\rangle, \langle p_1^2 p_2\rangle, \langle p_1 p_2^2\rangle\} \subset V(\mathsf{AG}(R))$. Then $\langle S\rangle = K_3$ in $\mathsf{AG}(R)$.

(iii) Without loss of generality, assume that $\alpha_i = \alpha_1$.

Let $S = \{\langle p_1^{\alpha_1 - 2} p_2^{\alpha_2}\rangle, \langle p_1^{\alpha_1 - 1} p_2^{\alpha_2}\rangle, \langle p_1^{\alpha_1}\rangle\} \subset V(\mathsf{AG}(R))$. Then $\langle S\rangle = K_3$ in $\mathsf{AG}(R)$.

(iv) Note that $S = \{\langle p_2^{\alpha_2} p_3^{\alpha_3} \cdots p_k^{\alpha_k}\rangle, \langle p_1^{\alpha_1} p_3^{\alpha_3} \cdots p_k^{\alpha_k}\rangle, \langle p_1^{\alpha_1} p_2^{\alpha_2} p_4^{\alpha_4} \cdots p_k^{\alpha_k}\rangle\} \subset V(\mathsf{AG}(R))$.

Then $\langle S\rangle = K_3$ in $\mathsf{AG}(R)$.

Conversely, it is enough to show that for the following cases:

(a) $n = p_1^{\alpha_1}$ $(\alpha_1 \leq 4)$; (b) $p_1 p_2$; (c) either $p_1^2 p_2$ or $p_1 p_2^2$.

For (a), if $n = p^2$, then $\mathsf{AG}(R) = K_1$. If $n = p^3$, then $\mathsf{AG}(R) = K_2$. If $n = p^4$, then $\mathsf{AG}(R) = P_3$. For (b), if $n = p_1 p_2$, then $\mathsf{AG}(R) = K_2$. For (c), if either $n = p_1^2 p_2$ or $p_1^2 p_2$, then $\mathsf{AG}(R) = P_4$.

**Corollary 2.4** *Let* $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ *with* $p_1, p_2, \ldots, p_k$ *are distinct primes,* $\alpha_1, \alpha_2, \ldots, \alpha_k$ *are positive integers,* $R = \mathsf{Z}_n$ *where* $n \neq p$ *and* $p$ *is a prime. Then* $\mathsf{AG}(R)$ *contains a path if and only if* $n$ *is any one of the following:*

i. $n = p_1^{\alpha_1}$ $(\alpha_1 \leq 4)$

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ii. $n = p_1 p_2$
iii. $n$ is either $p_1^2 p_2$ or $p_1 p_2^2$.

Note that, a graph $G$ is said to be *unicyclic* if $G$ contains exactly one cycle. In the following theorem, we characterize when $\mathrm{AG}(R)$ is unicyclic.

**Theorem 2.5** *Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ with $p_1, p_2, \ldots, p_k$ are distinct primes, $\alpha_1, \alpha_2, \ldots, \alpha_k$ are positive integers, $R = Z_n$ where $n \neq p$ and $p$ is a prime. Then $\mathrm{AG}(R)$ is unicyclic if and only if either $n = p_1^5$ or $n = p_1 p_2 p_3$.*

*Proof*. Assume that either $n = p_1^5$ or $n = p_1 p_2 p_3$. Proof follows from the embedding. Conversely, assume that $\mathrm{AG}(R)$ is unicyclic.

**Case 1.** $n = p_1^{\alpha_1}$. If $n = p_1^{\alpha_1}, \alpha_1 \leq 4,$ then, by Corollary 2.4, $\mathrm{AG}(R)$ is a path, a contradiction. Hence, $n = p_1^{\alpha_1}$ with $\alpha_1 \geq 5$. For $n = p_1^{\alpha_1}, \alpha_1 \geq 6,$ consider the sets $S_1 = \{<p^{\alpha_1 - 1}>, <p^{\alpha_1 - 2}>, <p^{\alpha_1 - 3}>\}$ and $S_2 = \{<p^{\alpha_1 - 1}>, <p^{\alpha_1 - 2}>, <p^{\alpha_1 - 4}>\}$. Then $<S_1> = <S_2> = K_3$ in $\mathrm{AG}(R)$, a contradiction. Hence, in this case, $n = p_1^5$.

**Case 2.** $k = 2$.

For any $n \in \mathbb{N}$ with $k = 2$, then $n$ is any one of the following cases:

(a) $p_1 p_2$; (b) either $p_1^2 p_2$ or $p_1 p_2^2$; (c) $p_1^2 p_2^2$; (d) either $\alpha_1 \geq 3$ or $\alpha_2 \geq 3$.

For (a) and (b), $\mathrm{AG}(R)$ are path, a contradiction.

(c) Let $S_1 = \{<p_1 p_2>, <p_1^2>, <p_2^2>\}$ and $S_2 = \{<p_1^2 p_2>, <p_1^2>, <p_2^2>\}$.

Then $<S_1> = <S_2> = K_3$ in $\mathrm{AG}(R)$, a contradiction.

(d) Without loss of generality, assume that $\alpha_1 \geq 3$. Let $S_1 = \{<p_1^{\alpha_1 - 2} p_2^{\alpha_2}>, <p_1^{\alpha_1 - 1} p_2^{\alpha_2}>, <p_1^{\alpha_1 - 1}>\}$ and $S_2 = \{<p_1^{\alpha_1}>, <p_1^{\alpha_1 - 2} p_2^{\alpha_2}>, <p_1^{\alpha_1 - 1} p_2^{\alpha_2}>\}$. Then $<S_1> = <S_2> = K_3$ in $\mathrm{AG}(R)$, a contradiction.

**Case 3.** $k = 3$.

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When $n = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$ and $\alpha_i \geq 2$ for some $i, 1 \leq i \leq 3$. Without loss of generality, assume that $\alpha_i = \alpha_1$. Let $S_1 = \{< p_1^{\alpha_1} p_2^{\alpha_2} >, < p_1^{\alpha_1 - 1} p_2^{\alpha_2} p_3^{\alpha_3} >, < p_1^{\alpha_1 - 1} p_3^{\alpha_3} >\}$ and $S_2 = \{< p_1^{\alpha_1} p_3^{\alpha_3} >, < p_1^{\alpha_1 - 1} p_2^{\alpha_2} p_3^{\alpha_3} >, < p_1^{\alpha_1 - 1} p_2^{\alpha_2} >\}$. Then $< S_1 > = < S_2 > = K_3$ in $\mathrm{AG}(R)$, a contradiction. Hence, in this case, $n = p_1 p_2 p_3$.

**Case 4.** $k \geq 4$.

Let $S_1 = \{< \frac{n}{p_1} >, < \frac{n}{p_2} >, < \frac{n}{p_3} >\}$ and $S_2 = \{< \frac{n}{p_1} >, < \frac{n}{p_2} >, < \frac{n}{p_4} >\}$. Then $< S_1 > = < S_2 > = K_3$ in $\mathrm{AG}(R)$, a contradiction.

Note that, a graph $G$ is a *claw-free* if $G$ does not have the claw $K_{1,3}$ as the induced subgraph. In the following theorem, we characterize when $\mathrm{AG}(R)$ is a claw-free graph.

**Theorem 2.6** *Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ with $p_1, p_2, \ldots, p_k$ are distinct primes, $\alpha_1, \alpha_2, \ldots, \alpha_k$ are positive integers, $R = \mathbb{Z}_n$ where $n \neq p$ and $p$ is a prime. Then $\mathrm{AG}(R)$ is a claw-free graph if and only if $n$ is any one of the following:*

i. $n = p_1^{\alpha_1}$ and $\alpha_1 \leq 5$;
ii. $n$ is either $p_1 p_2$ or $p_1^2 p_2$ or $p_1 p_2^2$ or $p_1^{\alpha_1} p_2^{\alpha_2}$ with $\alpha_i \geq 3$ for some $i$;
iii. $n = p_1 p_2 p_3$.

*Proof*. Proofs of (i), (ii) and (iii) are trivial.Conversely assume that $\mathrm{AG}(R)$ is a claw-free graph. It is enough to show that for the following cases, $\mathrm{AG}(R)$ is not a claw-free graph.

(a) $n = p_1^{\alpha_1}$ where $\alpha_1 \geq 6$; (b) $n = p_1^2 p_2^2$; (c) $p_1^{\alpha_1} p_2^{\alpha_2}$ where $\alpha_i \geq 3$ for some $i$;

(d) $n = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{a_3}$ where $\alpha_i \geq 2$ for some $i$; (e) $n = p_1^{\alpha_1} \ldots p_k^{a_k}$ where $k \geq 4$.

(a) Let $S = \{p_1, p_1^2, p_1^3, p_1^{\alpha_1 - 1}\}$. Then $< S > = K_{1,3}$ in $\mathrm{AG}(R)$.

(b) Let $S = \{< p_1 >, < p_1^2 >, < p_2 >, < p_1^2 p_2 >\}$. Then $< S > = K_{1,3}$ in $\mathrm{AG}(R)$.

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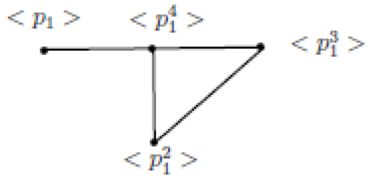
(c) Without loss of generality, assume that $\alpha_1 \geq 3$. Let $S = \{p_1, p_1^2, p_1^3, p_1^{\alpha_1 - 1} p_2^{\alpha_2}\}$. Then $< S >= K_{1,3}$ in $AG(R)$.

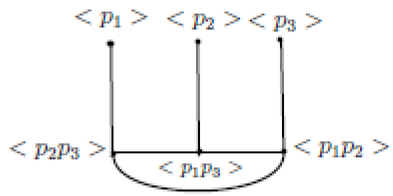
(d) Without loss of generality, assume that $\alpha_1 \geq 2$. Let $S = \{< p_1 >, < p_2 >, < p_3 >, < p_1^{\alpha_1 - 1} p_2^{\alpha_2} p_3^{\alpha_3} >\}$. Then $< S >= K_{1,3}$ in $AG(R)$.

(e) Let $S = \{p_2^{\alpha_2} \ldots p_k^{\alpha_k} >, < p_1^{\alpha_1} p_2^{\alpha_2} p_5^{\alpha_5} \ldots p_k^{\alpha_k} >, < p_1^{\alpha_1} p_3^{\alpha_3} p_5^{\alpha_5} \ldots p_k^{\alpha_k} >, < p_1^{\alpha_1} p_4^{\alpha_4} p_5^{\alpha_5} \ldots p_k^{\alpha_k} >\}$. Then $< S >= K_{1,3}$ in $AG(R)$.

Converse part is trivial.




$\mathbb{AG}(R)$ of $\mathbb{Z}_{p_1^5}$




$\mathbb{AG}(R)$ of $\mathbb{Z}_{p_1p_2p_3}$

**Theorem 2.7** *Let* $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ *with* $p_1, p_2, \ldots, p_k$ *are distinct primes,* $\alpha_1, \alpha_2, \ldots, \alpha_k$ *are positive integers,* $R = \mathbb{Z}_n$ *where* $n \neq p$ *and* $p$ *is a prime. Then* $AG(R)$ *is outerplanar if and only if* $n$ *is any one of the following:*

- $n = p_1^{\alpha_1}$ where $\alpha_1 \leq 6$
- $n = p_1^{\alpha_1} p_2^{\alpha_2}$ where $\alpha_1 \leq 2$ and $\alpha_2 \leq 2$
- $n = p_1^{\alpha_1} p_2^{\alpha_2}$ with either $\alpha_1 = 3$ and $\alpha_2 = 1$ or $\alpha_1 = 1$ and $\alpha_2 = 3$
- $n = p_1 p_2 p_3$.

*Proof.* Assume that $AG(R)$ is outerplanar.

**Case 1.** Suppose $n = p_1^{\alpha_1}$ where $\alpha_1 \geq 7$.

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Note that $p_1^{\alpha_1-3}.p_1^{\alpha_1-4} \geq p_1^{\alpha_1}$. Let $S=\{<p_1^{\alpha_1-1}>,<p_1^{\alpha_1-2}>,<p_1^{\alpha_1-3}>,<p_1^{\alpha_1-4}>\}$. Then $<S>=K_4$ in $\mathsf{AG}(R)$, a contradiction.

**Case 2.** Suppose $n=p_1^{\alpha_1}p_2^{\alpha_2}$ with either $\alpha_1=3$ and $\alpha_2\geq 2$ or $\alpha_1\geq 2$ and $\alpha_2=3$;

Without loss of generality assume that $\alpha_1=3$ and $\alpha_2\geq 2$.

Let $S=\{p_1, p_1^2p_2^{\alpha_2}, p_1^3, p_2^{\alpha_2}, p_1p_2^{\alpha_2}\}$. Then $<S>=K_{2,3}$ in $\mathsf{AG}(R)$, a contradiction.

**Case 3.** Suppose $n=p_1^{\alpha_1}p_2^{\alpha_2}$ where $\alpha_i\geq 4$ for some $i$.

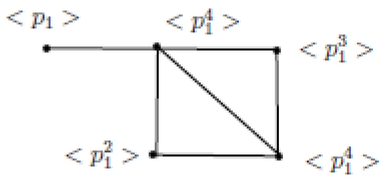
Without loss of generality assume that $\alpha_1\geq 4$. Let $S=\{p_1^{\alpha_1-1}, p_1^{\alpha_1-1}p_2^{\alpha_2}, p_1^{\alpha_1-2}p_2^{\alpha_2}, p_1^{\alpha_1-3}p_2^{\alpha_2}\}$. Then $<S>=K_4$ in $\mathsf{AG}(R)$, a contradiction.

**Case 4.** Suppose $n=p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}$ with $\alpha_i\geq 2$ for some $1\leq i\leq 3$;

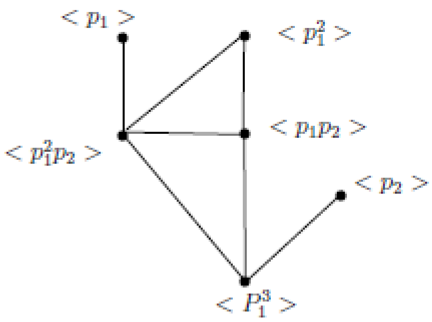
Without loss of generality assume that $\alpha_1\geq 2$. Let $S=\{<p_1^{\alpha_1}>,<p_1^{\alpha_1}p_2^{\alpha_2}>,<p_1^{\alpha_1}p_3^{\alpha_3}>,<p_2^{\alpha_2}p_3^{\alpha_3}>,<p_1^{\alpha_1-1}p_2^{\alpha_2}p_3^{\alpha_3}>\}$. Then $<S>$ contains a $K_{2,3}$ as a subgraph in $\mathsf{AG}(R)$, a contradiction.

**Case 5.** Suppose $k\geq 4$.

Let $S=\{<\frac{n}{p_1}>,<\frac{n}{p_2}>,<\frac{n}{p_3}>,<\frac{n}{p_4}>\}$. Then $<S>=K_4$ in $\mathsf{AG}(R)$, a contradiction.




$\mathbf{AG}(R) of \mathbb{Z}_{p_1^6}$




$\mathbf{AG}(R) of \mathbb{Z}_{p_1^3p_2}$

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