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Magic sum spectra of ladder graphs

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For a positive integer k, a graph $G=(V,E)$ is Z_k-magic if there exists a labeling $f: E(G) \to Z_k - \{0\}$ such that the induced vertex sum $f^+ : V(G) \to Z_k$ defined by $f^+(v) = \sum_{uv \in E(G)} f(uv)$ is a constant r is called a magic sum index. For fix integer k, the magic sum spectrum of G with respect to Z_k is the set of all magic sum indices r and it is denoted by $I_k(G)$. In this paper we obtained the integer magic spectra of certain classes of ladder graphs, möbius ladder graphs and some corono of ladder graphs.

Keywords: Z_k-magic, magic sum spectra, ladder graph, möbius ladder, corono graph.
Subject Classification: 05C78.

1 Introduction

A *graph labeling* is an assignment of integers to the vertices (or) edges or both subject to the certain constrains. Most graph labeling methods trace their origin to one introduced by Alex Rosa[4] in 1967. Since Sedl$\grave{a}\check{c}$ ek[5] introduced the notation of magic valution of a graph $G=(V,E)$ is a bijection from $V \cup E$ to $1,2,\cdots,|V \cup E|$ such that for all edge xy, $f(x)+f(y)+f(xy)$ is a constant called *magic constant* of f. A positive integer K is said to be magic constant of G. Later Ringel and Llado' rediscovered this notation and called it *edge magic labeling*.

For any positive integer $k \geq 2$, let $Z_k = (Z_k, +, 0)$ to be the additive abelian group of integer congruences modulo k with identity 0. We call a graph G to be Z_k magic if their exist a labeling $f: E(G) \to Z_k - \{0\}$ such that the vertex labeling f^+ defined as $f^+(v) = \sum_{uv \in E} f(uv) = r$ taken overall edges uv incident at v is a constant where r is called magic sum index of G,under the labeling of f, which follows R.P. Stanley[6].For a fixed k, $I_k(G)$ is the set of all magic sum indices r such that G is Z_k magic with an index r. We call $I_k(G)$ the magic sum spectrum, or the index set of G with respect to Z_k [7].

In this paper we obtained the magic sum spectra of some classes of ladder, Möbius ladder, and corona of ladder graphs which are finite simple connected graph, with vertexset V and edge set E and *Ladder graph* L_n is a planar undirected graph

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with $2n$ vertices and $n+2(n-1)$ edges. This graph can be obtained as the cartesian product of two path graphs one of the which has only one edge such that $L_{n,1} = P_n * P_1$. Adding two more crossed edges connecting the four (degree-two) vertices of a ladder graph produces a cublic circulant graph, the *Moböius ladder* M_n, is a(non-planar) with an even number n of vertices from an n-cycle by adding edges(rungs) connecting opposite pair of vertices in the cycle. Mobius ladders were named and first studied by Guy and Harary[3]. Specially M_8 is called the klaws wagnar graph. It play an important role in the theory of graph minors. A detailed survey on labeling of graphs, one can refer [1].

Walba, Richards and Haltiwagnar (1982) first synthesized molecular structures in the form of a mobius ladder and since then this structure has been of interest in chemistry and chemical stereography especially in the view of ladder like form of DNA-moleules with this application in the mind flapen(1989) studies the mathematical symetries of embedding of Möbius ladder in R^3. Möbius ladder have also been used in computer science as a part of integer programming.

Observation 1.1

$$I_k(L_n) = \begin{cases} \mathbb{Z}_k^* & \text{if } n=1 \text{ and } k \geq 2 \\ 2\mathbb{Z}_k^* & \text{if } n=2 \text{ and } k \geq 3 \\ \{0\} & \text{if } n=3 \text{ and } k=3 \\ \mathbb{Z}_k - 2 & \text{if } n=3 \text{ and } k=4 \\ \mathbb{Z}_k & \text{if } n=3 \text{ and } k \geq 5 \end{cases}$$

Theorem 1.2 *For* $n \geq 4, k \geq 5, I_k(L_n) = \mathbb{Z}_k$ *where* L_n *is the Ladder graph.*

Proof. Let the vertex set $V(L_n) = \{v_1, v_2, \ldots, v_{2n}\}$ and the edgeset $E(L_n) = E_1 \cup E_2$ where $E_1 = \{v_x v_{x+1} : 1 \leq x \leq n-1$ and $n+1 \leq x \leq 2n-1\}, E_2 = \{v_y v_{n+y} : 1 \leq x \leq n-1\}$ for t,k are positive integers under the function $f_t : E(L_n) \to \mathbb{Z}_k^*$

Case 1. For $1 \leq x \leq n-1$.

$$f_t(e) = \begin{cases} t & \text{if } e \in E_1 \\ k-1 & \text{if } e \in E_2 \text{ and } y = 1, n \\ k-t-1 & \text{if } e \in E_2 \text{ and } 2 \leq y \leq n-1 \end{cases}$$

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For each $v_i, 1 \le i \le 2n$, the induced map $f_t^+ = k+t-1 \equiv I_1(\bmod k)$ where $I_1 = \{0,1,2,\ldots,k-3\} \subset I_K(L_n)$.

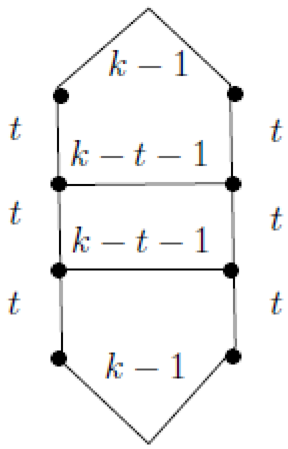
Case 2. For $t = k-1, t = k$.

$$f_t(e) = \begin{cases} t-2 & if \quad e \in E_1 \\ 1 & if \quad e \in E_2 \text{ and } y = 1, n \\ k-t+3 & if \quad e \in E_2 \text{ and } 2 \le y \le n-1 \end{cases}$$

the induced map $f_t^+(v_i) = f_t^+(v_n) = f_t^+(v_{n+1}) = f_t^+(v_{n+1}) = t-1 \equiv I_2(\bmod k)$ where $I_2 = \{k-2, k-1\} \subset I_k(L_n)$. For $2 \le i \le n-1$ and $n+2 \le i \le 2n-1$, $f_t^+(v_i) = k+t-1 \equiv I_2(\bmod k)$ where $I_2 = \{k-2, k-1\} \subset I_k(L_n)$. In the above two cases $I_k(L_n) = I_1 \cup I_2 = \{0,1,\ldots,k-2,k-1\} = Z_k, n \ge 4, k \ge 4$.

Example 1.3 *Consider the ladder graph L_4 with 10 edges. When $k = 5$, the magicsum spectra of L_4 is* $\{0,1,\cdots k-3\}$.

By case (1),



Z_k Magic labelling of L_4

By case (2), we obtain the remaining magic indices $\{k-2, k-1\}$

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Observation 1.4

$$I_k(M_n) = \begin{cases} \mathbb{Z}_k^* & if \quad n=1, \ \forall\, k \\ \mathbb{Z}_k^* & if \quad n=1, \ k=2 \\ \mathbb{Z}_k & if \quad n=2, \ \forall\, k>2 \end{cases}$$

Theorem 1.5 *For* $n \geq 4, k \geq 5, I_k(M_n) = \mathbb{Z}_k$ *where* M_n *is the Möbius Ladder graph.*

Proof. Let the vertex set $V(M_n) = \{v_1, v_2, v_3 \cdots v_{2n}\}$ and the edgeset $E(L_n) = E_1 \cup E_2$ where $E_1 = \{v_x v_{x+1} : 1 \leq x \leq 2n-1$ and $v_{2n}v_1$, $E_2 = \{v_y v_{n+y} : 1 \leq x \leq n\}$ for t, k are positive integers under the function $f_t : E(M_n) \to \mathbb{Z}_k^*$.

Case 1. For $1 \leq t \leq k-1$.

$$f_t(e) = \begin{cases} t & if \quad e \in E_1 \\ k-t & if \quad e \in E_2 \end{cases}$$

For each $v_i, 1 \leq i \leq 2n,$ the induced map $f_t^+(v_i) = k + t \equiv I_1 (\bmod k)$ where $I_1 = \{1, 2 \cdots, k-1\} \subset I_K(M_n)$.

Case 2. For $1 \leq t \leq k-1$.

$$f_t(e) = \begin{cases} t & if \quad e \in E_1 \\ k-2t & if \quad e \in E_2 \end{cases}$$

the induced map $f_t^+(v_i) = k \equiv I_2 (\bmod k)$.where $I_2 = \{0\} \subset I_k(M_n)$. In the above two cases $I_k(M_n) = I_1 \bigcup I_2 = \{0, 1, 2, \cdots, k-1\} = \mathbb{Z}_k, n \geq 4, k \geq 4$.

Example 1.6 *Consider the Mobious ladder* M_4 *with 12 edges. When* $k = 5,$ *the magic sum spectrum of* M_4 *is* $\{1, 2, \ldots, k-1\}$. *By case (1)*

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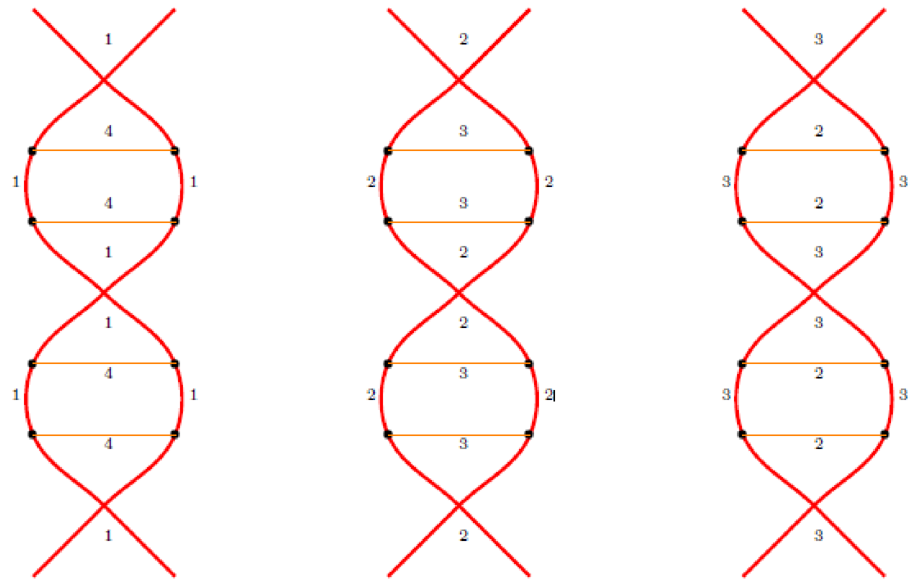
$\mathbb{Z}_5$ magic labeling of M_4



$f_1^+(v_i) \equiv 1 \mod 5$ $f_1^+(v_i) \equiv 2 \mod 5$ $f_1^+(v_i) \equiv 3 \mod 5$

Example 1.7 *Consider the Möbius-kantor graph M_8 with 24 edges. When $k=5$ the Magic spectrum of M_8 is* $\{1,2,\cdots,k-1\}$. *By case (1)*

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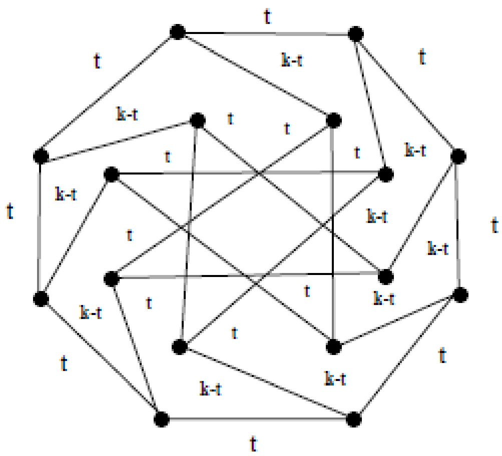

Mobius -Kantorgraph Z_5 Magic labeling.

Z_k -Magic labeling of corona graph

A new simple operation(non-commutative) on two graphs called their corona was first introduced by Frutch and Frank Harary(1970)[2]. Corona of graphs G with H, denoted by GeH is the graph with $V(GeH)=V(G)\cup\bigcup_{i\in v(G)}V(H)$ and $E(GeH)=E(G)\cup\bigcup_{i\in v(G)}E(H_i)\cup\{(i,u_i):i\in V(G)$ and $u_i\in V(H_i)\}$ In 1979, Frutch shown corona is graceful. In the following Frutch idea we assigning the Z_k magic labeling for corona of $L_n emk_1$ and $M_n emk_1$. The graph $Gemk_1$ is the graph obtained from the graph G by adding m number of pendent vertices to every vertex of G when $m=1$, $Gemk_1$ is known as corona of G.

Theorem 2.1 *For* $n\geq 4$ $k\geq 5, I_k(L_n ek_1)=$ Z_k^*

Proof. Let us take the vertex set $V(L_n ek_1)=V_1\cup V_2$ where $V_1=\{v_i:1\leq i\leq 2n\}, V_2=\{v_{2n+j}:1\leq i\leq 2n\}$ and the edge set $E(L_n ek_1)=E_1\cup E_2$ where

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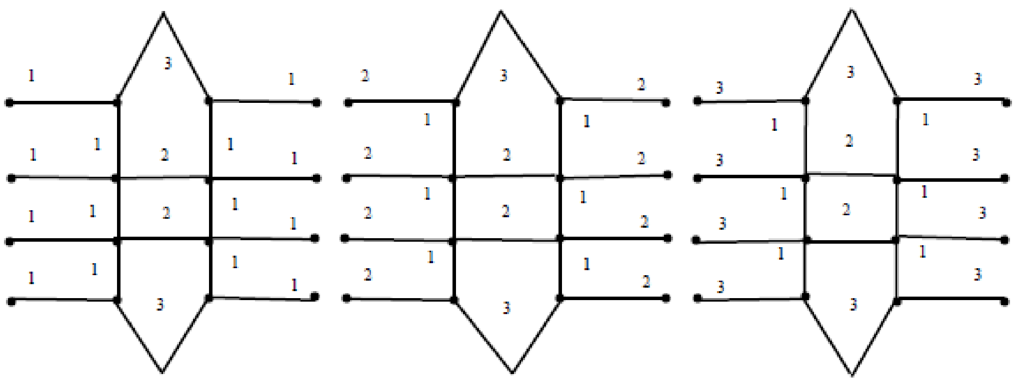
$E_1 = \{v_x v_{x+1} : 1 \le x \le n-1$ and $n+1 \le i \le 2n-1\} E_2 = \{v_y v_{y+1} : 1 \le y \le n-1\} E_3 = \{v_z v_{2n+z} : 1 \le z \le 2n\}$.For t,k are positive integers and define the function $f_t : E(L_n \odot k_1) \to Z_k^*$.

For $1 \le t \le k-1$

$$f_t(e) = \begin{cases} 1 & if \quad e \in E_1 \\ k-1 & if \quad e \in E_2 \text{ and } y = 1, n \\ k-t & if \quad e \in E_2 \text{ and } 2 \le y \le n-1 \\ t & if \quad e \in E_3 \end{cases}$$

For each $v_i, 1 \le i \le 4n$, the induced map $f_t^+(e) = k + t \equiv l (\bmod k)$ where $l = \{1, 2, \cdots k-1\}$. Hence $I_k(L_n \odot k_1) = Z_k^*$.

Example 2.2 *Consider the corona graph* $(L_4 \odot K_1)$ *for* $n = 4, k = 4$, *magicsum spectra of this corona is* Z_4^*. *By above theorem,*



$L_4 \odot k_1$

Theorem 2.3 *For* $n \ge 4 \quad k \ge 5 \quad I_k(M_n \odot k_1) = Z_k^*$

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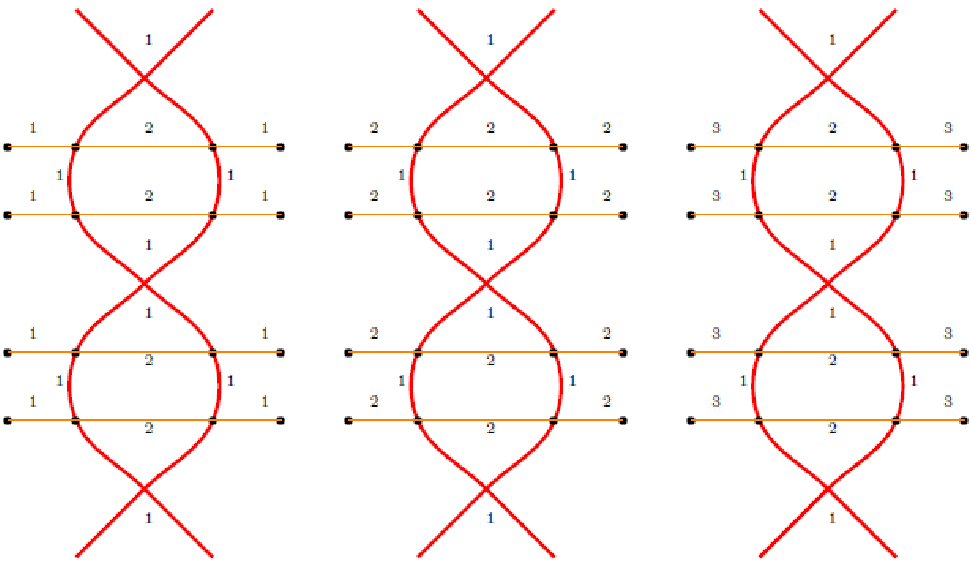
Proof. Let us take the vertex set $V(M_n \text{e} k_1) = V_1 \cup V_2$ where $V_1 = \{v_i : 1 \le i \le 2n\}$, $V_2 = \{v_{2n+j} : 1 \le i \le 2n\}$ and the edge set $E(M_n \text{e} k_1) = E_1 \cup E_2$ where $E_1 = \{v_x v_{x+1} : 1 \le x \le n-1 and n+1 \le i \le 2n-1\}$ $E_2 = \{v_y v_{y+1} : 1 \le y \le n-1\}$ $E_3 = \{v_z v_{2n+z} : 1 \le z \le 2n\}$. For $1 \le t \le k-1$,

$$f_t(e) = \begin{cases} 1 & if \quad e \in E_1 \\ k-2 & if \quad e \in E_2 \\ t & if \quad e \in E_3 \end{cases}$$

For each $v_i, 1 \le i \le 4n$, the induced map $f_t^+(e) = k + t \equiv \text{l}(\text{mod}\, k)$ where $\text{l} = \{1, 2, \cdots k-1\}$. Hence $I_k(M_n \text{e} k_1) = \text{Z}_k^*$.

Example 2.4 *Consider the corona graph* $(M_n \text{e} K_1)$ *for* $n = 4, k = 4$, *By applying above theorem,for* $1 \le t \le 3$ *we can obtain* $I_k(M_4 \text{e} k_1) = \text{Z}_4^*$ *as follows.*



$f_1^+(v_i) \equiv 1 \mod 4$ $\qquad$ $f_2^+(v_i) \equiv 2 \mod 4$ $\qquad$ $f_3^+(v_i) \equiv 3 \mod 4$

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