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k-Distance Signed Total Domination Number of Graphs

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Abstract

Let G be a finite and simple graph with the vertex set $V=V(G)$ of order n and edge set $E=E(G)$. If v is a vertex of a graph G, the open k-neighborhood of v, denoted by $N_k(v)$. A function $f:V(G)\to\{-1,+1\}$ is a k-distance non-negative signed total dominating function (k-DNNSTDF) of a graph G, if for every vertex $v\in V$, $f(N_k(v))=\sum_{u\in N_k(v)} f(u)\geq 0$. The k-distance non-negative signed total domination number (k-DNNSTDN) of a graph G equals the minimum weight of a k-DNNSTDF of G, denoted by $\gamma_{k,st}^{NN}(G)$. We study some properties of k-DNNSTDN in graphs and some families of graphs such as cycles, paths, complete graphs, star graphs and wheel graphs which admit 2-DNNSTDF.

Keywords: signed total dominating function, k-distance non-negative signed total dominating function.

1 *Introduction*

Let G be a finite and simple graph with the vertex set $V=V(G)$ of order n and edge set $E=E(G)$. If v is a vertex of a graph G, the open k-neighborhood of v, denoted by $N_k(v)$. $\delta_k(G)=min\{|N_k(v)|;v\in V\}$ and $\Delta_k(G)=max\{|N_k(v)|;v\in V\}$.

In 1995, J.E.Dunbar et al. defined signed dominating function. A function $f:V\to\{-1,+1\}$ is a *signed dominating function* of G, if for every vertex $v\in V$, $f(N[v])\geq 1$. The *signed domination number*, denoted by $\gamma_s(G)$, is the minimum weight of a signed dominating function on G JED.

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In 2001, Bohdan zelinka and Liberec introduced the concept of signed total domination function. A function $f: V \to \{-1,+1\}$ is a *signed total dominating function* of G, if for every vertex $v \in V$, $f(N(v)) \geq 1$. The *signed total domination number*, denoted by $\gamma_{st}(G)$, is the minimum weight of a signed total dominating function on G BOH01.

In 2013 ZhongshengHuang, Zhongsheng Huang et al. introduced the concept of on non-negative signed domination in graphs. A function $f: V \to \{-1,+1\}$ is a *non-negative signed dominating function* of G, if for every vertex $v \in V$, $f(N[v]) \geq 1$. The *non-negative signed domination number*, denoted by $\gamma_s^{NN}(G)$, is the minimum weight of a non-negative signed dominating function on G.

In this paper, we introduced the concept of k-distance non-negative signed total dominating function. A function $f: V(G) \to \{-1,+1\}$ is a k-distance non-negative signed total dominating function (k-DNNSTDF) of a graph G, if for every vertex $v \in V$, $f(N_k(v)) = \sum_{u \in N_k(v)} f(u) \geq 0$. The k-distance non-negative signed total domination number (k-DNNSTDN) of a graph G equals the minimum weight of a k-DNNSTDF of G, denoted by $\gamma_{k,st}^{NN}(G)$. We study some properties of k-DNNSTDN in graphs and some families of graphs such as cycles, paths, complete graphs, star graphs and wheel graphs which admit 2-DNNSTDF.

2 *Main results*

In this section, we obtain some properties of k-DNNSTDN in graphs.

Theorem 1 *Let G be a graph of order n. If* $\gamma_{k,st}^{NN}(G) = n$ *, then* $G \approx \overline{K_n}$.

Theorem 2 *For any graph G with maximum degree* Δ *and minimum degree* δ*, we have*

$$\gamma_{st}^{NN}(G) \geq \frac{\delta - \Delta}{\Delta + \delta} n.$$

Theorem 3 *Let* $n \geq 3$ *be an integer. Then the cycle* C_n *admits 2-DNNSTDF with* $\gamma_{2,st}^{NN}(C_n) \leq 0$ *when n is even and* $\gamma_{2,st}^{NN}(C_n) \leq 1$ *when n is odd.*

Lemma 1 *Let* $n \geq 3$ *be an odd integer. Then the path* P_n *admits 2-DNNSTDF with* $\gamma_{2,st}^{NN}(P_n) \leq 1$.

Lemma 2 *Let* $n \geq 4$ *be an even integer. Then the graph* P_n *admits 2-DNNSTDF with* $\gamma_{2,st}^{NN}(P_n) \leq 2$.

Form Lemma and Lemma , we can have the following theorem.

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Theorem 4 *Let $n\geq 3$ be an integer. Then the path P_n admits 2-DNNSTDF with $\gamma_{2,st}^{NN}(P_n)\leq 1$ when n is odd and $\gamma_{2,st}^{NN}(P_n)\leq 2$ when n is even.*

Lemma 3 *Let G be a connected graph of order n. Then $\gamma_{2,st}^{NN}(G)=n-2$ if and only if $G\approx P_3$ or C_3.*

Theorem 5 *The complete graph K_n $(n\geq 3)$ admits 2-DNNSTDF with $\gamma_{2,st}^{NN}(K_n)\leq 1$ when n is odd and $\gamma_{2,st}^{NN}(K_n)\leq 2$ when n is even.*

Lemma 4 *The star graph $K_{1,n}$ admits 2-DNNSTDF with $\gamma_{2,st}^{NN}(K_{1,n})\leq 1$ when n is even and $\gamma_{2,st}^{NN}(K_{1,n})\leq 2$ when is n odd.*

Theorem 6 *Let $n\geq 3$ be an integer. Then the wheel graph W_n admits 2-DNNSTDF with $\gamma_{2,st}^{NN}(W_n)\leq 1$ when n is even and $\gamma_{2,st}^{NN}(W_n)\leq 2$ when is n odd.*

Theorem 7 *The friendship graph T_n admit 2-DNNSTDF.*

Theorem 8 *For $n\geq 3$ be an integer. Then the helm graph H_n admits 2-DNNSTDF with $\gamma_{2,st}^{NN}(H_n)\leq 1$.*

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