Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

This volume makes available a record of articles presented in the conference. This volume contains the papers presented in the conference without any referring process.

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SUPREMACY LABELING OF SOME LADDER GRAPHS

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Abstract

The concept of labeling in graph theory is extended to a new horizon and hence introduced the supremacy labeling in [3]. Let G be a (p,q) graph. Its vertices are proper lableled and edges are lableled as the sum of the label values of vertices incident with that edge. The supremacy labeling is the function of V*(G) such that atleast one vertex of* G *must be labeled as zero and each non-zero labeled vertex must be adjacent to at least one zero labeled vertex provided that the edge set is properly labeled. A graph with supremacy labeling is the supremacy graph and the ordered pair of the least vertex label k of $V(G)$ and the least edge label l of $E(G)$ is the supremacy number of* G*, denoted by $\$(G) = (k, l)$. In this paper, the supremacy labeling of some ladder graphs are studied and the supremacy numbers are obtained.*

Keywords: Supremacy labeling, Supremacy graph, Supremacy number, some ladder graphs.

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1.Introduction

One of the most well structured graph modeling is the graph labeling which has unique impact in various fields. All graphs considered here are finite, simple, connected and undirected. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The concept of labeling was introduced by Rosa in 1967 and further developed by Graham and Sloane in 1980. There are many types of labeling; Gallian has given a survey of graph labeling [2]. Karonski, Luczak and Thomason initiated the study of proper labeling. Proper labeling is labeling the vertices (or edges) in such a way that no two adjacent vertices (or edges) receives the same label. Proper labeling has a great significance and is used in networking, circuit designing, designing of codes for radar, missile guidance, clustering, image segmentations, information security, coding theory, X-ray, crystallography, astronomy, representing chemical reactions and combinations. The structural arrangements of various objects and the techniques used thereby lead us to new inventions which in turn lead us to introduce the new labeling called Supremacy labeling [3]. A vital application of supremacy graph is in power or energy or resource allocation and distribution. The power distribution networks may be considered as a graph with supremacy labeling such that substations as zero labeled vertices, so that the power distribution is efficient. Supremacy labeling has a great significance and utilized in resource allocation of any network such as communication, transportation, security, chemical bond structures, etc.

In this paper, the supremacy labeling of some ladder graphs such as ladder, open ladder, Z-ladder or slash ladder, closed Z-ladder or closed slash ladder, circular ladder graphs are studied and the supremacy numbers are obtained.

2. *Preliminaries*

Definition 2.1:

Let $G = (V, E)$ be a simple graph with p vertices and q edges. The vertices of the set $V(G)$ are properly labeled and the edges of the set $E(G)$ are labeled with the sum of the label values of vertices incident with that edge. The supremacy labeling is the function of V(G) such that atleast one vertex must be labeled as zero and each non-zero labeled vertex must be adjacent to atleast one zero-labeled vertex provided that the edge set E is properly labeled. A graph G which admits supremacy labeling is called supremacy graph.

Definition 2.2:

The ordered pair (k, l) is said to be supremacy number of G denoted by $\$(G) = (k, l)$ if k is the least value of the vertex label and l is the least value of the edge label such that G is a supremacy graph with vertex labels $\{0,1,2, \dots, k\}$ and edge labels $\{1,2,3, \dots, l\}$.

Theorem 2.3:

Any graph G with $p \geq 4$ vertices is a supremacy graph if and only if there exists two vertices between any pair of vertices assigned with label k.

Definition 2.4:

The ladder graph $L_n, (n \geq 2)$ is defined as the Cartesian product of two paths P_2 and P_n, that is $L_n = P_2 \times P_n$.

Two paths $P_n(1) = u_1 u_2 u_3 \dots u_n$ and $P_n(2) = v_1 v_2 v_3 \dots v_n$ of a ladder L_n are called trails and the edges connecting two paths $u_i v_i: 1 \leq i \leq n$ are called rungs.

Definition 2.5:

Any ladder is said to be open (O) if there is no end rung $u_i v_i: i = 1, n$ and closed (Cl) if there are end rungs $u_i v_i: i = 1, n$.

Any open ladder can be converted into closed ladder by adding end rungs to it and any closed ladder can be converted into open ladder by removing end rungs from it.

Definition 2.6:

The Z-ladder (or Slash ladder) denoted by $ZL_n, (n \geq 2)$ is a graph defined as an ladder in which the rungs are joined from the vertex u_{i+1} to the vertex v_i.

Definition 2.7:

The circular ladder graph (or prism graph) $CL_n, (n \geq 3)$ is defined as the Cartesian product of path P_2 and cycle C_n, that is $CL_n = P_2 \times C_n$.

3. Main Results

Theorem 3.1:

The graph L_n is a supremacy graph when $n \equiv 1(mod\ 2)$ and $\$(L_n) = (3,5)$.

Proof:

Let $G = L_n$ be the graph.

Let $V[G] = \{u_i, v_i : 1 \le i \le n\}$ and

$E[G] = \{u_i u_{i+1} \cup v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$

When $n \equiv 1(mod\ 2)$

Let $f: V[G] \to \{0,1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 0 & i \equiv 1(mod\ 4) \\ 1 & i \equiv 2(mod\ 4) \\ 2 & i \equiv 3(mod\ 4) \\ 3 & i \equiv 0(mod\ 4) \end{cases} \quad for\ 1 \le i \le n$$

$$f(v_i) = \begin{cases} 2 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 2(mod\ 4) \\ 0 & i \equiv 3(mod\ 4) \\ 1 & i \equiv 0(mod\ 4) \end{cases} \quad for\ 1 \le i \le n$$

Then the induced edge labeling are

$$f(u_i u_{i+1}) = \begin{cases} 1 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 2,0(mod\ 4) \\ 5 & i \equiv 3(mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(v_i v_{i+1}) = \begin{cases} 5 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 2,0(mod\ 4) \\ 1 & i \equiv 3(mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(u_i v_i) \quad = \begin{cases} 2 & i \equiv 1(mod\ 2) \\ 4 & i \equiv 0(mod\ 2) \end{cases} \quad for\ 1 \le i \le n$$

Thus it is clear that the graph $L_n : n \equiv 1(mod\ 2)$ has the supremacy vertex labels $\{0,1,2,3\}$ and supremacy edge labels $\{1,2,3,4,5\}$.

Therefore, L_n is a supremacy graph.

Hence, the supremacy number is $\$(L_n) = (3,5)$.

Theorem 3.2:

The graph OL_n is a supremacy graph when $n = 4$ and $\$(OL_4) = (4,6)$**.**

Proof:

Let $G = OL_4$ be the graph.

Let $V[G] = \{u_i, v_i : 1 \le i \le 4\}$ and

$E[G] = \{u_i u_{i+1} \cup v_i v_{i+1} : 1 \le i \le 3\} \cup \{u_i v_i : 2 \le i \le 3\}$

Let $f: V[G] \to \{0,1,2,3,4\}$ be defined by

$$f(u_i) = f(v_i) = 0 \ \ for\ i \equiv 1(mod\ 3)$$

$$f(u_i) = \begin{cases} 1 & for\ i = 2 \\ 4 & for\ i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 3 & for\ i = 2 \\ 2 & for\ i = 3 \end{cases}$$

Then the induced edge labeling are

$$f(u_1 u_2) = 1, f(u_2 u_3) = f(v_2 v_3) = 5, f(u_3 u_4) = 4$$

$$f(v_1 v_2) = 3, f(v_3 v_4) = 2$$

$$f(u_i v_i) = \begin{cases} 4 & for\ i = 2 \\ 6 & for\ i = 3 \end{cases}$$

Thus it is clear that the graph OL_4 has the supremacy vertex labels $\{0,1,2,3,4\}$ and supremacy edge labels {1,2,3,4,5,6}.

Therefore, OL_n is a supremacy graph.

Hence, the supremacy number is $\$(OL_n) = (4,6)$.

Theorem 3.3:

The graph ZL_n is a supremacy graph when $n \equiv 2,3,0\ (mod\ 4)$ and

$$\$(L_n) = \begin{cases} (2,3) & for\ n = 2 \\ (3,4) & for\ n = 3 \\ (3,5) & for\ n \geq 4 \end{cases} \quad when\ \ n \equiv 2,3,0\ (mod\ 4).$$

Proof:

Let $G = ZL_n$ be the graph.

Let $V[G] = \{u_i, v_i: 1 \leq i \leq n\}$ and

$E[G] = \{u_i u_{i+1} \cup v_i v_{i+1}: 1 \leq i \leq n-1\} \cup \{u_{i+1} v_i: 1 \leq i \leq n-1\}$

Let $f: V[G] \rightarrow \{0,1,2,3\}$ be defined by

Case 1: When $n \equiv 2, 3\ (mod\ 4)$

$$f(u_i) = \begin{cases} 0 & i \equiv 1 (mod\ 4) \\ 1 & i \equiv 2 (mod\ 4) \\ 3 & i \equiv 3 (mod\ 4) \\ 2 & i \equiv 0 (mod\ 4) \end{cases} \qquad for\ 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 2 & i \equiv 1 (mod\ 4) \\ 0 & i \equiv 2 (mod\ 4) \\ 1 & i \equiv 3 (mod\ 4) \\ 3 & i \equiv 0 (mod\ 4) \end{cases} \qquad for\ 1 \leq i \leq n$$

Then the induced edge labeling are

$$f(u_i u_{i+1}) = \begin{cases} 1 & i \equiv 1\ (mod\ 4) \\ 4 & i \equiv 2\ (mod\ 4) \\ 5 & i \equiv 3\ (mod\ 4) \\ 2 & i \equiv 0\ (mod\ 4) \end{cases} \qquad for\ 1 \leq i \leq n-1$$

$$f(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1\ (mod\ 4) \\ 1 & i \equiv 2\ (mod\ 4) \\ 4 & i \equiv 3\ (mod\ 4) \\ 5 & i \equiv 0\ (mod\ 4) \end{cases} \qquad for\ 1 \leq i \leq n-1$$

$$f(u_{i+1} v_i) = 3 \quad for\ 1 \leq i \leq n-1$$

Case 2: When $n \equiv 0\ (mod\ 4)$

$$f(u_i) = \begin{cases} 2 & i \equiv 1 (mod\ 4) \\ 0 & i \equiv 2 (mod\ 4) \\ 1 & i \equiv 3 (mod\ 4) \\ 3 & i \equiv 0 (mod\ 4) \end{cases} \qquad for\ 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 3 & i \equiv 1 (mod\ 4) \\ 2 & i \equiv 2 (mod\ 4) \\ 0 & i \equiv 3 (mod\ 4) \\ 1 & i \equiv 0 (mod\ 4) \end{cases} \qquad for\ 1 \leq i \leq n$$

Then the induced edge labeling are

$$f(u_iu_{i+1}) = \begin{cases} 2 & i \equiv 1\ (mod\ 4) \\ 1 & i \equiv 2\ (mod\ 4) \\ 4 & i \equiv 3\ (mod\ 4) \\ 5 & i \equiv 0\ (mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(v_iv_{i+1}) = \begin{cases} 5 & i \equiv 1\ (mod\ 4) \\ 2 & i \equiv 2\ (mod\ 4) \\ 1 & i \equiv 3\ (mod\ 4) \\ 4 & i \equiv 0\ (mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(u_{i+1}v_i) = 3\ \ for\ 1 \le i \le n-1$$

Thus from the above two cases it is clear that the graph $ZL_n: n \equiv 2,3,0(mod\ 4)$ has the supremacy vertex labels {0,1,2,3} and supremacy edge labels {1,2,3,4,5}.

Therefore, ZL_n is a supremacy graph.

Hence, the supremacy number is $\$(L_n) = \begin{cases} (2,3) & for\ n = 2 \\ (3,4) & for\ n = 3 \\ (3,5) & for\ n \ge 4 \end{cases}$ $when\ \ n \equiv 2,3,0\ (mod\ 4)$.

Theorem 3.4:

The graph $ClZL_n$ is a supremacy graph for $n = 3, n \equiv 0\ (mod\ 2)$ and

$$\$(ClZL_n) = \begin{cases} (3,4) & for\ n = 2 \\ (4,6) & for\ n = 3 \\ (4,5) & for\ n \equiv 0(mod\ 2) \end{cases}$$

Proof:

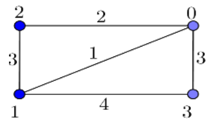
Let $G = ClZL_n$ be the graph.

Let $V[G] = \{u_i, v_i: 1 \le i \le n\}$ and

$E[G] = \{u_iu_{i+1} \cup v_iv_{i+1}: 1 \le i \le n-1\} \cup \{u_{i+1}v_i: 1 \le i \le n-1\} \cup \{u_iv_i: i = 1, n\}$

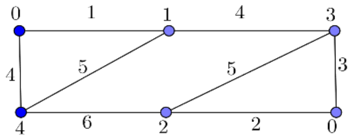
Case 1: When $n = 2$

$f: V[G] \to \{0,1,2,3\}$ is labeled as shown in the figure



Case 2: When $n = 3$

$f: V[G] \to \{0,1,2,3,4\}$ is labeled as shown in the figure



Case 3: When $n \equiv 0\ (mod\ 2)$

Let $f: V[G] \to \{0,1,2,3,4\}$ be defined by

$$f(u_1) = f(v_n) = 4$$

$$f(u_i) = \begin{cases} 3 & i \equiv 1(mod\ 4) \\ 0 & i \equiv 2(mod\ 4) \\ 2 & i \equiv 3(mod\ 4) \\ 1 & i \equiv 0(mod\ 4) \end{cases} \quad for\ 2 \le i \le n$$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 2(mod\ 4) \\ 0 & i \equiv 3(mod\ 4) \\ 2 & i \equiv 0(mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

Then the induced edge labeling are

$$f(u_iu_{i+1}) = \begin{cases} 4 & i = 1, i \equiv 0(mod\ 4) \\ 3 & i = 3, i \equiv 1(mod\ 4) \\ 2 & i \equiv 2(mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(v_iv_{i+1}) = \begin{cases} 4 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 2,0(mod\ 4) \\ 2 & i \equiv 3(mod\ 4) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(v_iv_{i+1}) = \begin{cases} 4 & n \equiv 0(mod\ 4) \\ 5 & n \equiv 2(mod\ 4) \end{cases} \quad for\ i = n-1$$

$$f(u_{i+1}v_i) = \begin{cases} 1 & i \equiv 1(mod\ 2) \\ 5 & i \equiv 0(mod\ 2) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(u_1v_1) = 5$$

$$f(u_nv_n) = \begin{cases} 4 & when\ n \equiv 2(mod\ 4) \\ 5 & when\ n \equiv 0(mod\ 4) \end{cases}$$

Thus from the above three cases it is clear that the graph $ClZL_n$ has the supremacy vertex labels $\{0,1,2,3\}$ when $n = 2, \{0,1,2,3,4\}$ when $n = 3\ and\ n \equiv 0(mod\ 2)$ and the supremacy edge labels $\{1,2,3,4\}$ when $n = 2, \{1,2,\dots,6\}$ when $n = 3, \{1,2,\dots,5\}$ when $n \equiv 0(mod\ 2)$.

Therefore, $ClZL_n$ is a supremacy graph.

Hence, the supremacy number is $\$(ClZL_n) = \begin{cases} (3,4) & for\ n = 2 \\ (4,6) & for\ n = 3 \\ (4,5) & for\ n \equiv 0(mod\ 2) \end{cases}$

Theorem 3.5:

The graph CL_n is a supremacy graph for $n \equiv 0(mod\ 4)$ and $\$(CL_n) = (3,5)$.

Proof:

Let $G = CL_n$ be the graph.

Let $V[G] = \{u_i, v_i : 1 \le i \le n\}$ and

$$E[G] = \{u_iu_{i+1} \cup v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_nu_1\} \cup \{v_nv_1\} \cup \{u_iv_i : 1 \le i \le n\}$$

Let $f: V[G] \to \{0,1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 0 & i \equiv 1(mod\ 4) \\ 2 & i \equiv 2(mod\ 4) \\ 1 & i \equiv 3(mod\ 4) \\ 3 & i \equiv 0(mod\ 4) \end{cases} \quad for\ 1 \le i \le n$$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 2(mod\ 4) \\ 0 & i \equiv 3(mod\ 4) \\ 2 & i \equiv 0(mod\ 4) \end{cases} \quad for\ 1 \le i \le n$$

Then the induced edge labeling are

$$f(u_iu_{i+1}) = \begin{cases} 2 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 0(mod\ 2) \\ 4 & i \equiv 1(mod\ 2) \end{cases} \quad for\ 1 \le i \le n-1$$

$$f(v_i v_{i+1}) = \begin{cases} 4 & i \equiv 1(mod\ 4) \\ 3 & i \equiv 0(mod\ 2) \\ 2 & i \equiv 3(mod\ 4) \end{cases} \quad for\ 1 \leq i \leq n-1$$

$$f(u_n u_1) = f(v_n v_1) = 3$$

$$f(u_i v_i) = \begin{cases} 1 & i \equiv 1(mod\ 2) \\ 5 & i \equiv 0(mod\ 2) \end{cases} \quad for\ 1 \leq i \leq n$$

Thus it is clear that the graph $CL_n : n \equiv 0(mod\ 4)$ has the supremacy vertex labels $\{0,1,2,3\}$ and supremacy edge labels $\{1,2,3,4,5\}$.

Therefore, CL_n is a supremacy graph.

Hence, the supremacy number is $\$(CL_n) = (3,5)$.

4. Conclusion

The ladder graphs can be constructed as supremacy graphs which have great significance and utilization in resource allocation. Hence obtained the supremacy number of some ladder graphs and will be further extended to other types of graphs.

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