Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

This volume makes available a record of articles presented in the conference. This volume contains the papers presented in the conference without any referring process.

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DEGREE SPLITTING OF STOLARSKY-3 MEAN LABELING OF GRAPHS

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ABSTRACT

A graph $G = (V, E)$ with p vertices and q edges is called a Stolarsky-3 mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,….,q+1 in such a way that when each edge e =uv is labeled with f=(e=uv)=$\left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$, then the edge labels are distinct. In this case f is called Stolarsky-3 mean labeling of G. If G be a graph with $V = S_1 \cup S_2 \cup \ldots \cup S_T$, where each S_i is a set of vertices having atleast two vertices and $T = V - \cup S_i$.The degree splitting graph of G by $DS(G)$ and is obtained from G by adding vertices $w_1, w_2, \ldots, w_n$ and joining w_i to each vertex of $S_i (1 \leq i \leq t)$. In this paper, we contribute some new results on Stolarsky-3 mean labeling of degree splitting graphs.

Keywords: Labeling, Stolarsky-3 Mean Labeling, Degree splitting graph.

AMS Subject Classification: 05C78

1. INTRODUCTION:

The graph considered here will be simple, finite and undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A. Gallian [1]. For all other standard terminology and notations we follow Harary[2].The concept of Mean labeling was introduced in [3]. Motivated by the above results and by the motivation of the authors we study the Stolarsky-3 Mean labeling on Degree Splitting graphs.In this paper we investigate Stolarsky-3 mean labeling behaviour of degree splitting graph of some graphs. We will provide brief summary of definitions and other information which are necessary for our present investigation.

Definition 1.1:

A graph $G = (V, E)$ with p vertices and q edges is called a Stolarsky-3 mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,….,q+1 in such a way that when each edge e =uv is labeled with f=(e=uv)=$\left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$, then the edge labels are distinct. In this case f is called Stolarsky-3 mean labeling of G

Definition 1.2:

Let $G = (V, E)$be a graph with$V = S_1 \cup S_2 \cup \ldots . \cup S_i \cup T$, where each $S_i$is a set of vertices having atleast two vertices and $T = V - \cup S_i$. The degree splitting graph of G is denoted by $DS(G)$ and is obtained from G by adding vertices$w_1, w_2, \ldots . w_t$and joining$w_i$to each vertex of$S_i (1 \leq i \leq t)$. The graph G and its degree splitting graph $DS(G)$ are given in figure:1.

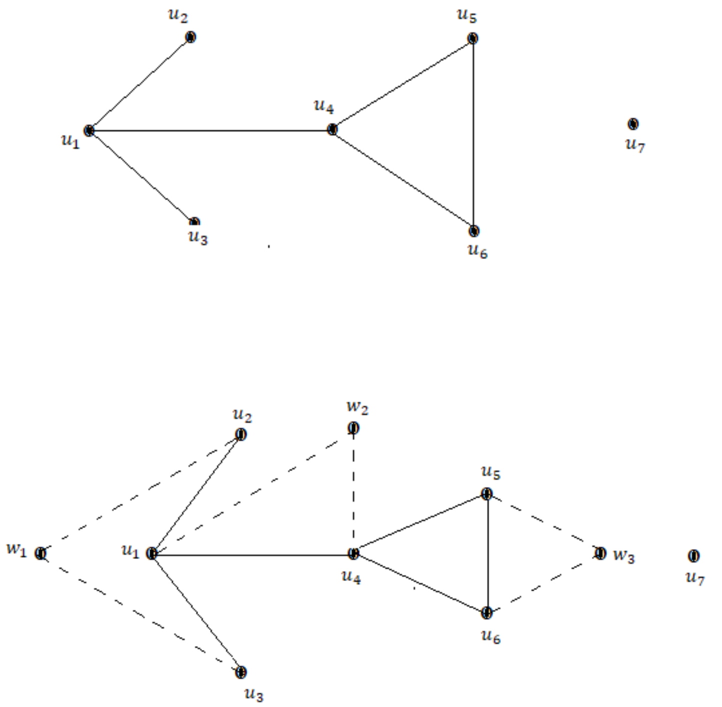


Figure: 1

Remark 1.3:

Any graph G is a subgraph of $DS(G)$. If G has atleast two vertices, then G contains atleast two vertices of the same degree. Hence $G = K_1$is the only graph such that $G = DS(G)$.

Theorem 1.4: Any path is a Stolarsky-3 mean graph.

Theorem 1.5: Any cycle is a Stolarsky-3 mean graph.

Theorem 1.6: If $n > 3$,$K_n$is not a Stolarsky-3 mean graph.

Theorem 1.7: Wheel is not Stolarsky-3mean graph.

Theorem 1.8: nK_3 is a Stolarsky-3 mean graph.

Remark 1.9: In general, $nDS(K_{1,m})$ is a Stolarsky-3 mean graph.

2. MAIN RESULTS:

Theorem: 2.1

$nDS(P_3)$ is a stolarsky-3 mean graph.

Proof:

The graph $DS(P_3)$ is shown figure: 2

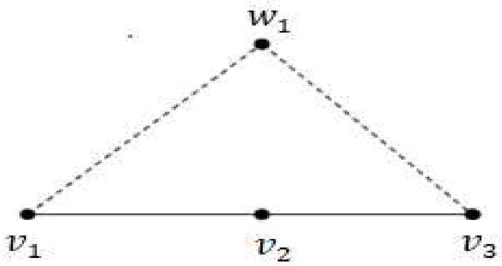


Figure: 2

Let $G = nDS(P_3)$, let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$ where $V_i = \{v_1{}^i, v_2{}^i, v_3{}^i, w_i, 1 \le i \le n\}$

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$$f(v_1{}^i) = 4i - 3, 1 \le i \le n, f(v_2{}^i) = 4i - 2, 1 \le i \le n$$

$$f(v_3{}^i) = 4i - 1, 1 \le i \le n, f(w_1{}^i) = 4i, 1 \le i \le n$$

Then the edges are labeled as

$$f(v_1{}^i v_2{}^i) = 4i - 3, 1 \le i \le n, f(v_2{}^i v_3{}^i) = 4i - 2, 1 \le i \le n$$

$$f(v_1{}^i w_i) = 4i - 1, 1 \le i \le n, f(v_3{}^i w_i) = 4i, 1 \le i \le n$$

Thus f provides a Stolarsky-3 mean labeling of G. Hence $nDS(P_3)$ is a Stolarsky-3 mean graph.

Example: 2.2 Stolarsky-3 mean labeling of $4DS(P_3)$ is given in the following figure: 3

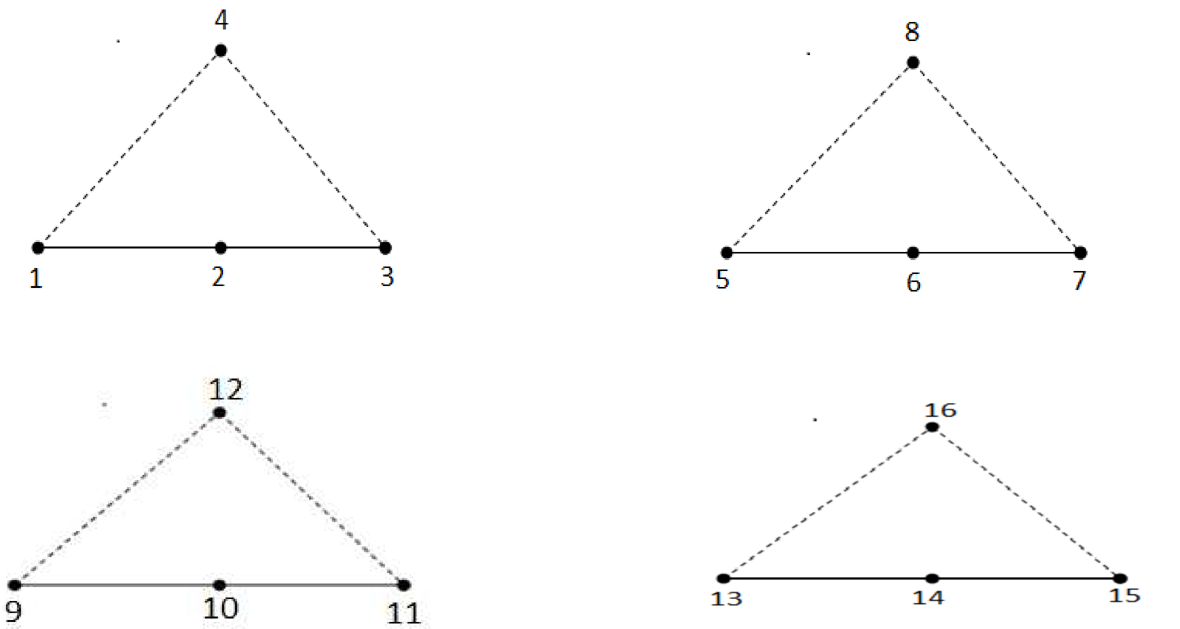


Figure: 3

Remark: 2.3

$nDS(P_4)$is a Stolarsky-3 mean graph

Theorem: 2.4

$nDS(P_3 \odot K_1)$is a Stolarsky-3 mean graph.

Proof:

The graph $DS(P_3 \odot K_1)$ is a Stolarsky-3 mean graph.

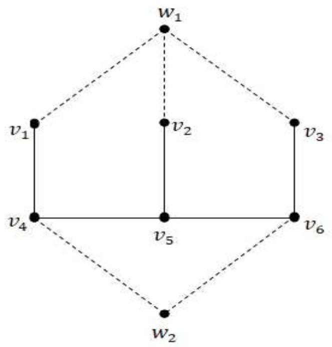


Figure: 4

Let $G = nDS(P_3 \odot K_1)$, let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$ where $V_i = \{{v_1}^i, {v_2}^i, {v_3}^i, {v_4}^i, {v_5}^i, {v_6}^i, {w_1}^i, {w_2}^i, 1 \leq i \leq n\}$

Define a function $f: V(G) \rightarrow \{1,2, \ldots, q+1\}$ by

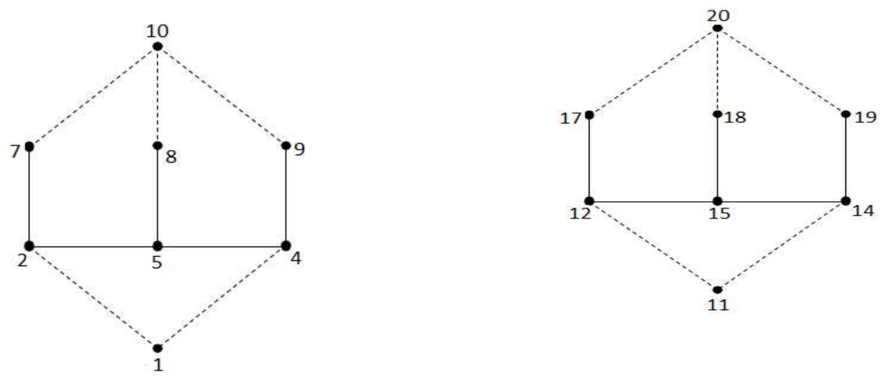
$$f({v_1}^i) = 10i - 3, 1 \leq i \leq n, f({v_2}^i) = 10i - 2, 1 \leq i \leq n$$
$$f({v_3}^i) = 10i - 1, 1 \leq i \leq n\ , f({v_4}^i) = 10i - 8, 1 \leq i \leq n$$
$$f({v_5}^i) = 10i - 5{,}1 \leq i \leq n\ , f({v_6}^i) = 10i - 6{,}1 \leq i \leq n$$
$$f({w_1}^i) = 10i, 1 \leq i \leq n\ , f({w_2}^i) = 10i - 9{,}1 \leq i \leq n$$

Then the edges are labeled as

$$f({v_1}^i {v_4}^i) = 10i - 5, 1 \leq i \leq n\ , f({v_4}^i {v_5}^i) = 10i - 7, 1 \leq i \leq n$$
$$f({v_2}^i {v_5}^i) = 10i - 4, 1 \leq i \leq n\ , f({v_2}^i {v_6}^i) = 10i - 6, 1 \leq i \leq n$$
$$f({v_3}^i {v_6}^i) = 10i - 3, 1 \leq i \leq n\ , f({v_4}^i {w_1}^i) = 10i - 9, 1 \leq i \leq n$$
$$f({w_1}^i {v_6}^i) = 10i - 8, 1 \leq i \leq n\ , f({v_1}^i {w_2}^i) = 10i - 2, 1 \leq i \leq n$$
$$f({w_2}^i {v_2}^i) = 10i - 9, 1 \leq i \leq n\ , f({w_2}^i {v_3}^i) = 10i, 1 \leq i \leq n$$

Thus f provides a Stolarsky-3 mean labeling for G. Hence $nDS(P_3 \odot K_1)$ is a Stolarsky-3 mean graph.

Example: 2.5 Stolarsky-3 mean labeling of $4DS(P_3 \odot K_1)$ is shown in figure: 5



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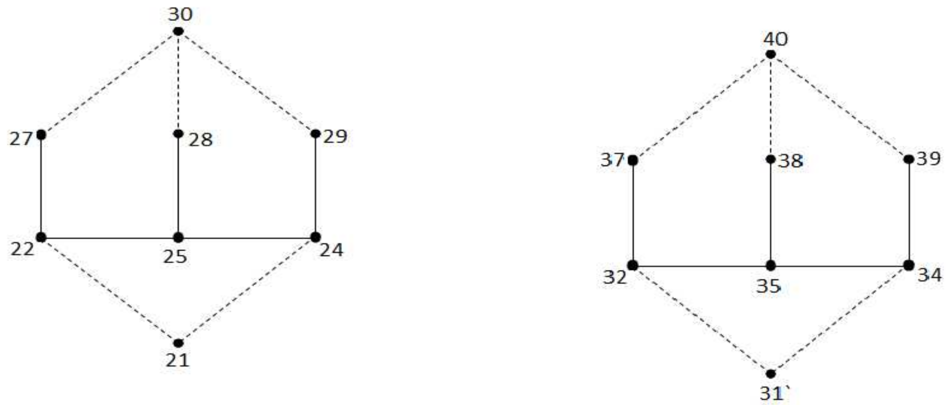

Figure: 5

Remark: 2.6

$nDS(P_3 \odot K_{1,2})$is a Stolarsky-3 mean graph.

Remark: 2.7

$nDS(P_3 \odot K_{1,3})$ is a Stolarsky-3 mean graph.

Theorem: 2.8

$nDS(K_{1,3})$ is a stolarsky-3 mean graph.

Proof:

The graph $DS(K_{1,3})$ is shown in figure: 6

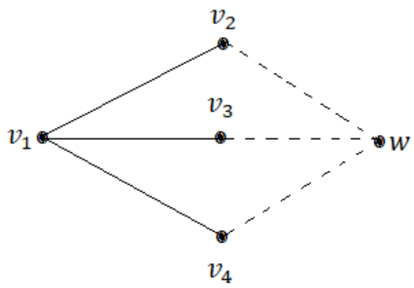

Figure: 6

Let $G = nDS(K_{1,3})$. Let the vertex set of Gbe $V = V_1 \cup V_2 \cup \ldots . \cup V_n$,
Where$V_i = \{v_1{}^i, v_2{}^i, v_3{}^i, v_4{}^i, w_i / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $nDS(K_{1,3})$.
Define a function $f: V(G) \to \{1,2, \ldots \ldots q+1\}$ by

$$f(v_1{}^i) = 6i - 5; 1 \le i \le n, f(v_2{}^i) = 6i - 4; 1 \le i \le n$$
$$f(v_3{}^i) = 6i - 3; 1 \le i \le n, f(v_4{}^i) = 6i - 1; 1 \le i \le n$$
$$f(w^i) = 6i; 1 \le i \le n$$

Then the edges are labeled as,

$$f(v_1{}^i v_2{}^i) = 6i - 5; 1 \le i \le n, f(v_1{}^i v_3{}^i) = 6i - 4; 1 \le i \le n$$
$$f(v_1{}^i v_4{}^i) = 6i - 3; 1 \le i \le n, f(v_2{}^i w^i) = 6i - 1; 1 \le i \le n$$
$$f(v_3{}^i w^i) = 6i - 2; 1 \le i \le n, f(v_4{}^i w^i) = 6i; 1 \le i \le n$$

Thus f provides a Stolarsky-3 mean labeling for G. Hence $nDS(K_{1,3})$ is a Stolarsky-3 mean graph.

Example: 2.9 Stolarsky-3 mean labelling of $4DS(K_{1,3})$ is shown in figure: 7.

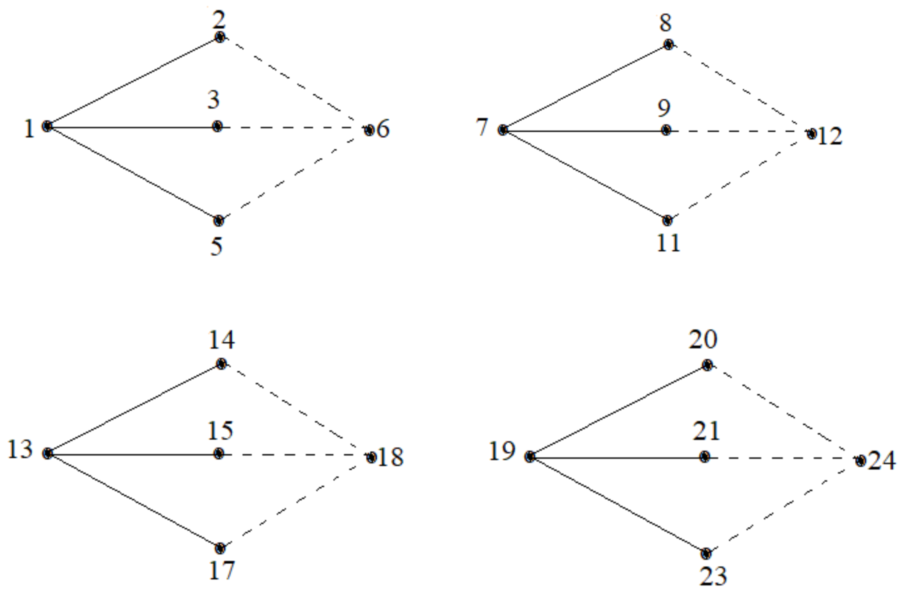

Figure: 7

Remark: 2.10

$nDS(K_{1,4})$is a Stolarsky-3 mean graph.

Theorm: 2.11

$nDS(C_4 \circ K_{1,3})$is a Stolarsky-3 mean graph.

Proof:

The graph$DS(C_4 \circ K_{1,3})$is shown in figure: 8

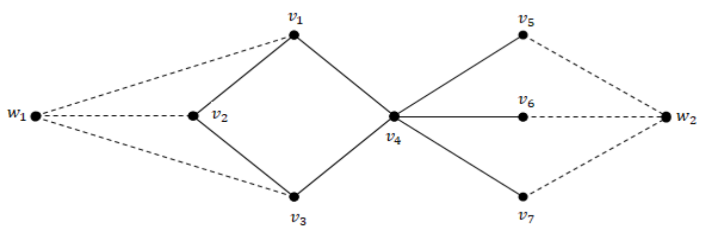

Figure: 8

Let $G = nDS(C_4 \circ K_{1,3})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots . \cup V_n$, Where $V_i = \{v_1{}^i, v_2{}^i, v_3{}^i, v_4{}^i, v_5{}^i, v_6{}^i, v_7{}^i, w_1{}^i, w_2{}^i / 1 \leq i \leq n\}$.

Define a function $f: V(G) \rightarrow \{1,2, \ldots \ldots q+1\}$ by

$$f(v_1{}^i) = 13i - 10; 1 \leq i \leq n, f(v_2{}^i) = 13i - 11; 1 \leq i \leq n$$

$$f(v_3{}^i) = 13i - 7; 1 \leq i \leq n, f(v_4{}^i) = 13i - 6; 1 \leq i \leq n$$

$$f(v_5{}^i) = 13i - 4; 1 \leq i \leq n, f(v_6{}^i) = 13i - 3; 1 \leq i \leq n$$

$f(v_7{}^i) = 13i - 2; 1 \leq i \leq n, f(w_1) = 13i - 12; 1 \leq i \leq n$
$f(w_1) = 13i; 1 \leq i \leq n$

Then the edges are labeled as,

$f(v_1{}^i v_2{}^i) = 13i - 10; 1 \leq i \leq n, f(v_2{}^i v_3{}^i) = 13i - 8; 1 \leq i \leq n$
$f(v_1{}^i v_4{}^i) = 13i - 7; 1 \leq i \leq n, f(v_3{}^i v_4{}^i) = 13i - 6; 1 \leq i \leq n$
$f(v_1{}^i w_1{}^i) = 13i - 11; 1 \leq i \leq n , f(v_2{}^i w_1{}^i) = 13i - 12; 1 \leq i \leq n$
$f(v_3{}^i w_1{}^i) = 13i - 9; 1 \leq i \leq n, f(v_4{}^i v_5{}^i) = 13i - 5; 1 \leq i \leq n$
$f(v_4{}^i v_6{}^i) = 13i - 4; 1 \leq i \leq n, f(v_4{}^i v_7{}^i) = 13i - 3; 1 \leq i \leq n$

Thus f provides a Stolarsky-3 mean labeling for G. Hence $nDS(C_4 \circ K_{1,3})$ is a Stolarsky-3 mean graph.

Example: 2.12 Stolarsky-3 mean labeling of $4DS(C_4 \circ K_{1,3})$is shown in figure: 9

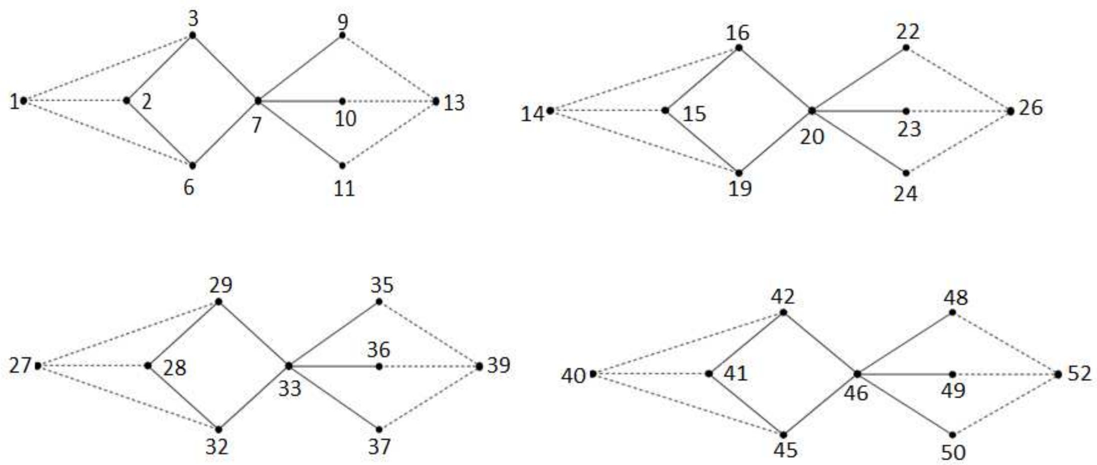

Figure: 9

Remark: 2.13

$nDS(C_4 \circ K_{1,4})$is a Stolarsky-3 mean graph.

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