Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

This volume makes available a record of articles presented in the conference. This volume contains the papers presented in the conference without any referring process.

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ARBITRARY SUPER SUBDIVISION OF ROOT SQUARE MEAN LABELING OF GRAPHS

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ABSTRACT

A graph G = (V, E) with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices x∈V with distinct labels f (x) from 1,2,……….. q+1 in such a way that when each edge e = uv is labeled with f(e) = $\left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the resulting edge labels are distinct. In this case f is called a Root Square Mean labeling of G. In this paper we investigate Arbitrary super subdivision concept to Root Square Mean Graphs.

Key words: Graph , Labeling , Root Square Mean labeling , Super subdivision of graphs , Arbitrary Super sub division of graphs , Path, Cycle, Triangular snake Quadrilateral snake , Alternate Triangular snake and Alternate Quadrilateral snake.

INTRODUCTION

The graph considered here will be finite, undirected and simple. The vertex set is denoted by V(G) and the edge set is denoted by E(G). For all detailed survey of graph labelling, we refer to Galian [1]. For all other standard terminology and notations we follow Harary[2] . G. Sethuraman and P. Selvaraju introduced a new method of construction called Super subdivision of a graph. In this paper we study the Root Square Mean labeling of arbitrary super subdivision of Path, Cycle, Triangular snake graph , Quadrilateral snake , Alternate Triangular snake and Alternate Quadrilateral snake graph.

Definition1.1 : A graph G = (V,E) with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices x∈V with distinct labels f (x) from 1,2,……….. q+1 in such a way that when each edge e = uv is labeled with f(uv) = $\left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case, f is called a Root Square Mean labeling of G.

Definition 1.2 : Let G be a graph . A graph H is called a **Super Subdivision** of G, if H is obtained from G by replacing every edge e_i of G by a Complete bipartite graph k_{2,m_i} for some m_i ,$1 \leq i \leq q$ is such a way that the ends of e_i are merged with two vertices part of k_{2,m_i} after removing the edge e_i from graph G.

Definition 1.3: A Super subdivision H of G is said to be an **Arbitrary Super subdivision** of G if every edge of G is replaced by an Arbitrary k_{2,m_i} where m_i may vary for each edge arbitrarily. It is denoted by ASS(G).

Theorem 1.4: Arbitrary Super subdivisions of paths are Root Square Mean graphs.

Proof: Let P_n be a Path with successive vertices $u_1, u_2, u_3, \dots u_n$. Let $e_i$denote the edge $u_i u_{i+1}$ of P_n for $1 \leq i \leq n-1$. Let H be an arbitrary super subdivision of a path P_n, where each edge e_i of P_n is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer ($m_1 \leq 4$).

$$V(H) = \{u_1, u_2, u_3, \dots., u_n, v_1, v_2, v_3, \dots, v_{m_1}, v_{m_1+1}, v_{m_1+2}, \dots, v_{m_1+m_2}, \dots,$$

$$\dots. v_{m_1+m_2+m_3+\dots+m_{n-1}}\}$$

Arbitrary super subdivision of P_n

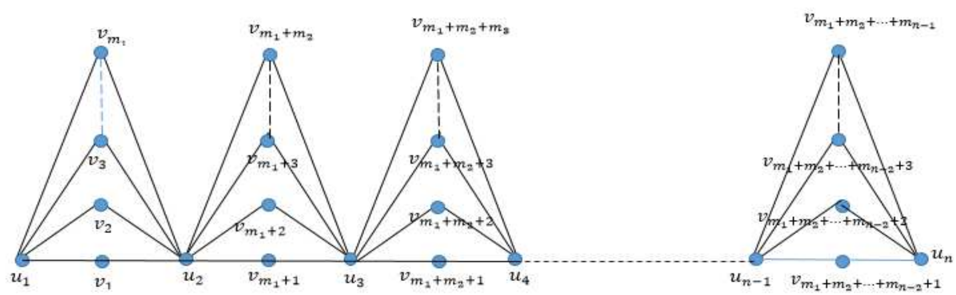

Figure :1

Here we consider two different cases.

Case (i) : $\boldsymbol{m_1 = m_2 \cdots = m_{n-1} = 3}$

An Arbitrary super subdivision of P_5 is given in the following figure

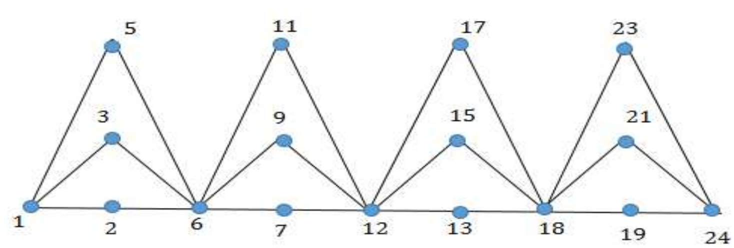

Figure : 2

Define a function $\varphi: V(H) \to \{1,2,3 \dots, q+1\}$ by

$\varphi(u_1) = 1$, $\varphi(u_i) = 6(i-1)$, $2 \leq i \leq n$

$\varphi(v_1) = 2\,, \quad \varphi(v_i) = 2i - 1\,, \quad 2 \le i \le m_1 + m_2 + \cdots + m_{n-1}.$

Then the edge labels are distinct. In this case H is a Root Square Mean graph.

Case (ii) $m_1 = m_2 = \cdots\cdots = m_{n-1} = 4$

An Arbitrary super subdivision of P_5 is given in the following figure

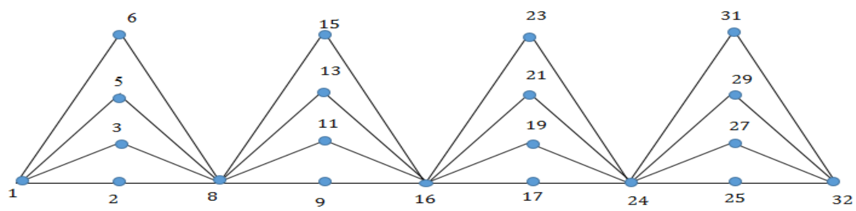

Figure: 3

Define $\varphi: V(H) \to \{1,2,3 \ldots, q+1\}$ by

$\varphi(u_1) = 1 \;;\; \varphi(u_i) = 8(i-1)\,, \; 2 \le i \le n \;;\; \varphi(v_1) = 2 \;;\; \varphi(v_i) = 2i - 1\,, \; 2 \le i \le 3$

$\varphi(v_4) = 6\,, \quad \varphi(v_i) = 2i - 1\,, \quad 5 \le i \le m_1 + m_2 + \cdots + m_{n-1}.$

Then the edge labels are distinct. From case (i) and case (ii) we can conclude that Arbitrary Super Subdivision of Paths are Root Square mean graphs.

Remarks 1.5: The above results is true for all values of n's and m_i's with condition $m_i \le 4$, $1 \le i \le n-1$.

Theorem 1.6: Arbitrary Super subdivisions of Cycles are Root Square Mean graphs.

Proof : Let C_n be a Cycle with consecutive vertices $u_1, u_2, u_3, \ldots u_n$. Let e_i denote the edge $u_{i-1}u_i$ of C_n for $1 \le i \le n-1$. Let H be an arbitrary super subdivision of a Cycle C_n, where each edge e_i of C_n is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer ($m_1 \le 4$).

$$V(H) = \{u_1, u_2, u_3, \ldots., u_n, v_1, v_2, v_3, \ldots, v_{m_1}, v_{m_1+1}, v_{m_1+2}, \ldots, v_{m_1+m_2}, \ldots,$$

$$\ldots. v_{m_1+m_2+m_3+\cdots+m_{n-1}}, v_{m_1+m_2+m_3+\cdots m_{n-1}} + 1$$

Arbitrary super subdivision of C_n

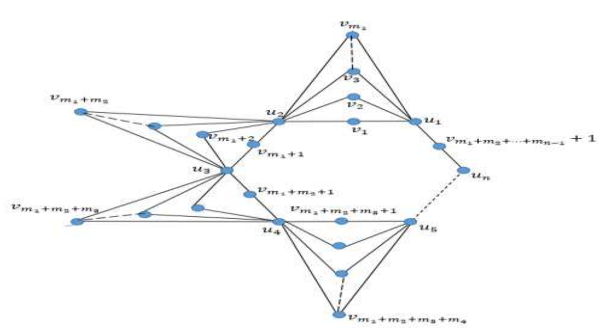

Figure :4

Case (i) : $m_1 = m_2 \cdots = m_{n-1} = 3$

An Arbitrary super subdivision of C_4 is given in the following figure

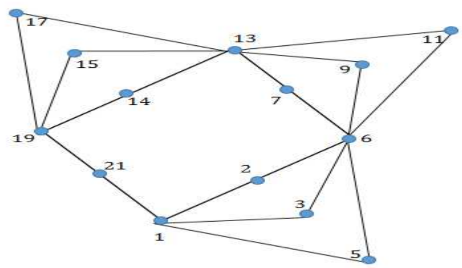

Figure : 5

Define a function $\varphi: V(H) \to \{1,2,3 \dots, q+1\}$ by

$\varphi(u_1) = 1$; $\varphi(u_i) = 6(i-1)$, $2 \le i \le n-2$

$\varphi(u_{n-1}) = \varphi(u_{n-2}) + 7$; $\varphi(u_n) = \varphi(u_{n-1}) + 6$

$\varphi(v_1) = 2$, $\varphi(v_i) = 2i - 1$, $2 \le i \le m_1 + m_2 + \cdots + m_{n-2}$

$\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-2}+1}) = \varphi(v_{m_1+m_2+m_3+\cdots m_{n-2}}) + 3$

$\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-2}+2}) = \varphi(u_{n-1}) + 2$

$\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-1}}) = \varphi(u_{n-1}) + 4$

$\varphi(v_{m_1+m_2+m_3+\cdots m_{n-1}} + 1) = \varphi(u_n) + 2$

Then the edge labels are distinct. In this case H is a Root Square Mean graph.

Case (ii) $m_1 = m_2 = \cdots\cdots = m_{n-1} = 4$

An Arbitrary super subdivision of C_5 is given in the following figure

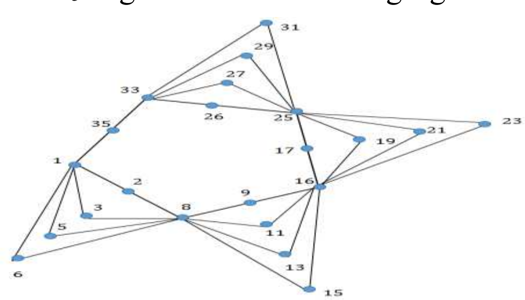

Figure : 6

Define $\varphi: V(H) \to \{1,2,3 \dots, q+1\}$ by

$\varphi(u_1) = 1$, $\varphi(u_i) = 8(i-1)$, $2 \le i \le n-2$

$\varphi(u_{n-1}) = \varphi(u_{n-2}) + 9$; $\varphi(u_n) = \varphi(u_{n-1}) + 8$; $\varphi(v_1) = 2$; $\varphi(v_4) = 6$

$\varphi(v_i) = 2i - 1$, $2 \le i \le 3$; $\varphi(v_i) = 2i - 1$,$5 \le i \le m_1 + m_2 + \cdots + m_{n-2}$

$\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-2}+1}) = \varphi(u_{n-1}) + 1$, $\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-2}+2}) = \varphi(u_{n-1}) + 2$

$\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-2}+3}) = \varphi(u_{n-1}) + 4$, $\varphi(v_{m_1+m_2+m_3+\cdots+m_{n-1}}) = \varphi(u_n) - 2$

$\varphi(v_{m_1+m_2+m_3+\cdots m_{n-1}} + 1) = \varphi(u_n) + 2$

Then the edge labels are distinct. From case (i) and case (ii) we can conclude that Arbitrary Super Subdivision of Cycles are Root Square mean graphs.

Remarks 1.7: The above results is true for all values of n's and m_i's with condition $m_i \le 4$, $1 \le i \le n-1$.

Theorem 1.8: Arbitrary Super subdivisions of Triangular snake T_n is a Root Square Mean graph.

Proof : Let T_n be the Triangular snake graph with the vertices $u_1, u_2, u_3, \ldots u_n$ & $v_1, v_2, v_3, \ldots, v_{n-1}$.

Let $u_i u_{i+1}, u_i v_i, v_i u_{i+1}, 1 \le i \le n-1$ be the edges of a Triangular snake T_n. Let H be an arbitrary super subdivision of T_n, where each edge e_i of T_n is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer.

$V(H) = \{u_1, u_2, u_3, \ldots \ldots, u_n, v_1, v_2, \ldots \ldots, v_{n-1}, w_{m_1}(k), w_{m_2}(k), \ldots, w_{m_{n-1}}(k), w_1, w_2, \ldots,$

$w_{m_1}, w_{m_1+1}, w_{m_1+2}, \ldots, w_{m_1+m_2}, w_{m_1+m_2+1}, \ldots, w_{m_1+m_2+m_3+\cdots+m_{2n-2}}\}$

Arbitrary Super Subdivision of T_n

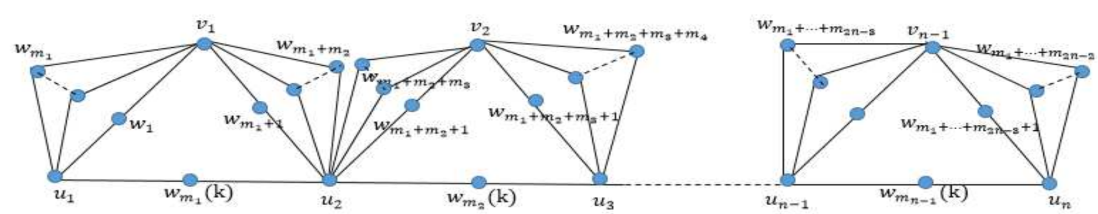

Figure : 7

When $\boldsymbol{m_1 = m_2 = \cdots\cdots = m_{2n-2} = 3}$

Arbitrary Super Subdivision of T_4 is given below

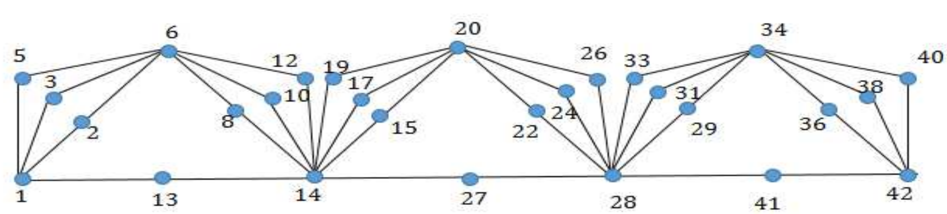

Figure : 8

Define a function $\varphi: V(H) \rightarrow \{1,2,3 \dots, q+1\}$ by

$\varphi(u_1) = 1$; $\varphi(u_i) = 14(i-1)$, $2 \leq i \leq n$

$\varphi(v_i) = 14i - 8$, $1 \leq i \leq n-1$; $\varphi(w_1) = 2$; $\varphi(w_2) = 3$

$\varphi(w_i) = \varphi(w_{i-1}) + 2$, $3 \leq i \leq m_1 + m_2 + \cdots + m_{2n-2}$ &

$i \neq m_1 + 1, (m_1 + m_2) + 1, \dots, (m_1 + m_2 + m_3 + \dots + m_{2n-3}) + 1$

$\varphi(w_{m_1+1}) = \varphi(w_{m_1}) + 3$; $\varphi\left(w_{\sum_{i=1}^{k} m_i + 1}\right) = \varphi\left(w_{\sum_{i=1}^{k} m_i} + 3\right)$ $k = 2,3, \dots, 2n-3$

$\varphi(w_{m_i}(k)) = 14i - 1$, $1 \leq i \leq n-1$

Then the edge labels are distinct.In this similar manner we can prove for all n's and m_i's ($m_1 \leq 4$). Hence Arbitrary Super subdivisions of Triangular snake T_n is a Root Square Mean graph.

Theorem 1.9: Arbitrary Super subdivisions of Quadrilateral snake Q_n is a Root Square Mean graph.

Proof : Let Q_n be the Quadrilateral snake graph with the vertices $u_1, u_2, u_3, \dots u_n$ & $v_1, v_2, v_3, \dots, v_{n-1}$ & $w_1, w_2, w_3, \dots, w_{n-1}$.

Let $u_i v_i, v_i w_i, w_i u_{i+1}$, $1 \leq i \leq n-1$ be the edges of a Quadrilateral snake Q_n .Let H be an arbitrary super subdivision of Q_n, where each edge e_i of Q_n is replaced by a complete bipartite graph k_{2,m_i} where m_i is any positive integer.

$V(H) = \{u_1, u_2, u_3, \dots \dots, u_n, v_1, v_2, \dots \dots, v_{n-1}, w_1, w_2, \dots, w_{n-1}, x_1, x_2, \dots, x_{m_1}, x_{m_1+1}$

$x_{m_1+2}, \dots, x_{m_1+m_2}, x_{m_1+m_2+1}, \dots, x_{m_1+m_2+m_3+\cdots+m_{n+5}}, x_{m_1}(k), x_{m_2}(k), \dots, x_{m_{n-1}}(k)\}$

Arbitrary Super Subdivision of Q_n is given below

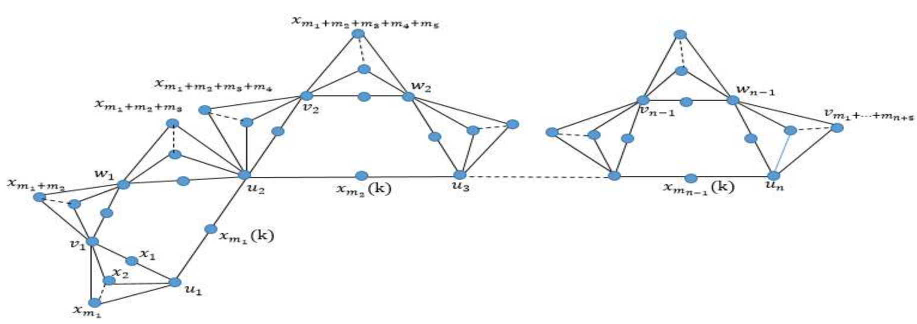

Figure : 9

When $\boldsymbol{m_1 = m_2 = \cdots\cdots = m_{n+5} = 3}$

Arbitrary Super Subdivision of Q_4 is given in the following figure

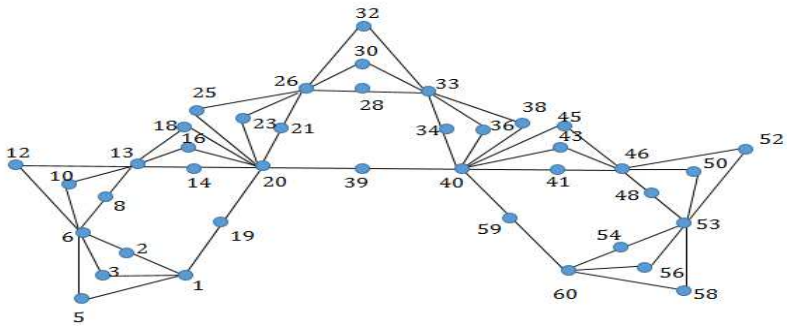

Figure :10

Define a function $\varphi: V(H) \rightarrow \{1,2,3 \ldots, q+1\}$ by

$\varphi(u_1) = 1$; $\varphi(u_i) = 20(i-1)$, $2 \le i \le n$; $\varphi(v_i) = 20i - 14$, $1 \le i \le n-1$

$\varphi(w_i) = 20i - 7$; $1 \le i \le n-1$, $\varphi(x_1) = 2$, $\varphi(x_2) = 3$

$\varphi(x_i) = \varphi(x_{i-1}) + 2$, $3 \le i \le m_1 + m_2 + \cdots + m_{n+5}$ &

$i \ne m_1 + 1, (m_1 + m_2) + 1, \ldots, (m_1 + m_2 + m_3 + \ldots + m_{n+4}) + 1$

$\varphi(x_{m_i}(k)) = 20i - 1$, $1 \le i \le n-1$

$\varphi\left(x_{\sum_{i=1}^{k} m_i + 1}\right) = \varphi\left(w_{\sum_{i=1}^{k} m_i} + 3\right)$ $if\ k = 1,3, \ldots, n+4$ and

$\left(x_{\sum_{i=1}^{k} m_i + 1}\right) = \varphi\left(w_{\sum_{i=1}^{k} m_i} + 2\right)$ $if\ k = 2,5,8,11,14 \ldots$

Then the edge labels are distinct. In this similar manner we can prove for all n's and m_i's ($m_1 \le 4$). Hence Arbitrary Super subdivisions of Quadrilateral snake Q_n is a Root Square Mean graph.

Remarks 1.10: Arbitrary Super subdivisions of Alternate Triangular snake $A(T_n)$ and Alternate Quadrilateral snake $A(Q_n)$ are Root Square Mean graphs for all n's and m_i's ($m_1 \le 4$).

CONCLUSION : All graphs are not Root Square mean graphs. It is very interesting to investigate graphs which admit Root Square mean labeling. In this paper we proved that , Arbitrary super subdivision of Path, Cycle, Triangular snake graph , Quadrilateral snake , Alternate Triangular snake and Alternate Quadrilateral snake graph are Root Square mean graphs.

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