Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

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Dr. T. Tamizh Chelvam

Dr. T. Asir

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SOME STRONG FORMS OF CONTINUOUS FUNCTIONS IN NANO TOPOLOGY

M.Davamani Christober[1], A.Vinith Mala[2]

[1,2]Department of Mathematics,
The American College,
Madurai-625002, Tamilnadu
[1]Email:christober.md@gmail.com
[2]Email:vinithmala1986@gmail.com

Abstract

The objective of this paper is to introduce some stronger types of continuous functions called strongly nano$\alpha\hat{g}$-continuous function, perfectly nano$\alpha\hat{g}$- continuous function and contra nano $\alpha\hat{g}$-continuous function. Further some properties and characterizations of these functions are obtained. Also their relationships with other continuous functions are investigated.

Keywords: Nano $\alpha\hat{g}$-closed set, Strongly nano $\alpha\hat{g}$- continuous function, perfectly nano $\alpha\hat{g}$-continuous function and contra nano $\alpha\hat{g}$- continuous function.

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1. Introduction

Njastad[8] and Levine[7] have introduced α-open sets and generalized closed sets respectively. Lellis Thivagar[4] introduced Nano Topological space with respect to a subset of X of an universe which is defined in terms of lower and upper approximations of X. Davamani Christober and Vinith Mala[2] have defined $\alpha\hat{g}$-closed sets and $\alpha\hat{g}$-continuous function in Nano Topological space.

In this paper, we investigate some stronger forms of continuous functions namely strongly nano $\alpha\hat{g}$-continuous function, perfectly nano $\alpha\hat{g}$- continuous function and contra nano $\alpha\hat{g}$-continuous function. In addition some properties and characterizations of these functions are examined as well as their relationships with existing continuous functions are explored.

2. Preliminaries

Definition 2.1[4]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

i. The lower approximation of X with respect to R is denoted by $L_R(X)$ and is defined as $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, R(*x*) denotes the equivalence class determined by *x*.

ii. The upper approximation of X with respect to R is denoted by $U_R(X)$ and is defined as $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

iii. The boundary region of X with respect to R is denoted by $B_R(X)$ and is defined as $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[4]: Let U be an universe. R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

i. U and $\phi \in \tau_R(X)$

ii. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

iii. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to *X*. We call $(\mathrm{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Definition 2.3[4]: Let $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and is denoted by $N\operatorname{int}(A)$. The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$.

Definition 2.4[2]: A subset A of a space $(U, \tau_R(X))$ is called nano $\alpha\hat{g}$-closed if $N_\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano $\hat{g}$-open in $(U, \tau_R(X))$. The complement of nano $\alpha\hat{g}$-closed set is nano $\alpha\hat{g}$-open set.

Definition 2.5: A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be

i. Nano-continuous[5] on *U* if the inverse image of every nano open set in *V* is nano open in *U*.

ii. Nano α-continuous [5] on *U* if the inverse image of every nano open set in *V* is nano α-open in *U*.

iii. Nano *g*-continuous [1] on *U* if the inverse image of every nano open set in *V* is nano g-open in *U*.

iv. Nano α-generalized continuous [10] on *U* if the inverse image of every nano open set in *V* is nano α generalized open in *U*.

v. Nano $\hat{g}$-continuous [3] on U if the inverse image of every nano open set in *V* is nano $\hat{g}$-open in U.

vi. Nano $\alpha\hat{g}$-continuous [2] if the inverse image of every nano closed set in V is nano $\alpha\hat{g}$-closed in U.

vii. Nano $\alpha\hat{g}$-irresolute [2] if the inverse image of every nano $\alpha\hat{g}$-closed set in V is nano $\alpha\hat{g}$-closed in U.

viii. Nano strongly continuous [6] if $f^{-1}(A)$ is nano clopen in U for every subset A in V.

ix. Nano Perfectly continuous [6] if $f^{-1}(A)$ is nano clopen in U for every nano open set A in V.

x. Nano contra continuous [6] if the inverse image of every nano open set in V is nano closed in U.

3. Strongly Nano $\alpha\hat{g}$-continuous functions

In this section, we introduce the following definition.

Definition 3.1: A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$is said to be strongly nano $\alpha\hat{g}$-continuous if $f^{-1}(B)$ is nano closed in U for every nano $\alpha\hat{g}$-closed set B in V.

Example 3.2: Let $U = \{p, q, r, s\}$, $U/R = \{\{p\}, \{q, r\}, \{s\}\}$ and $X = \{q, s\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{s\}, \{q, r\}, \{q, r, s\}\}$. Also let $V = \{a, b, c, d\}, {}^{V}/_{R'} = \{\{a, b\}, \{c\}, \{d\}\}$, $Y = \{a, c\} \subseteq V$. Then nano $\alpha\hat{g}$-closed sets of V are$\{V, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Define a function $f: U \to V$ by $f(p) = d, f(q) = a$, $f(r) = a$ and $f(s) = a$. Then inverse image of every nano $\alpha\hat{g}$-closed sets in V is nano closed in U. Hence f is strongly nano$\alpha\hat{g}$-continuous.

Theorem 3.3: A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is strongly nano $\alpha\hat{g}$-continuous if and only if $f^{-1}(B)$ is nano open in $(U, \tau_R(X))$ for every nano $\alpha\hat{g}$-open set B in$(V, \tau_R(Y))$.

Theorem 3.4: Every strongly nano$\alpha\hat{g}$-continuous function is

i. Nano continuous and hence nano generalized continuous and nano α-continuous.
ii. Nano $\alpha\hat{g}$-continuous and hence nano αg- continuous and nano gs-continuous
iii. Nano $\hat{g}$-continuous.

Proof: Follows from the definitions.

Remark 3.5: The following example shows that the implications in theorem 3.4 are not reversible in general.

Example 3.6: Let $U = \{a, b, c, d\}, X = \{a, c\}$ and $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. Also $V = \{p, q, r, s\}$, $Y = \{q, s\}$ and $\tau_R(Y) = \{V, \emptyset, \{s\}, \{q, r\}, \{q, r, s\}\}$. Define $f: U \to V$ by$f(a) = q, f(b) = r, f(c) = s$ and $f(d) = p$. Here f is nano continuous, nano $\alpha\hat{g}$-continuous, nanoα-continuous, nano$g$-continuous, nano$\hat{g}$-continuous, nano gs-continuous and nano αg-continuous but not strongly nano $\alpha\hat{g}$-continuous.

Theorem 3.7 Every strongly nano continuous function is strongly nano $\alpha\hat{g}$ - continuous function

Proof: Follows from the definitions.

Remark 3.8: The following example shows that the converse of theorem 3.7 need not be true.

Example 3.9: In example 3.2, f is strongly nano$\alpha\hat{g}$-continuous but not strongly nano continuous.

Theorem 3.10: Every strongly nano $\alpha\hat{g}$- continuous function is nano $\alpha\hat{g}$-irresolute.

Proof: Follows from the definitions.

Corollary 3.11: Every strongly nano continuous function is nano$\alpha\hat{g}$-irresolute.

Proof follows from theorem 3.7 and theorem 3.10.

Remark3.12: The converse of the theorem 3.10 need not be true as the following example shows.

Example 3.13: $U = \{a, b, c, d\}$with $X = \{a, c\} \subseteq U$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. Also let $V = \{p, q, r, s\}$, ${}^{V}/_{R'} = \{\{q, r\}, \{p\}, \{s\}\}$, $Y = \{q, s\} \subseteq V$. Then nano $\alpha\hat{g}$-closed sets of V are $V, \emptyset, \{p\}, \{p, q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, [p, q, s\}, \{p, r, s\}$. Define $f: U \to V$ by $f(a) = s, f(b) = r, f(c) = q, f(d) = p$. Here f is nano$\alpha\hat{g}$-irresolute but not strongly nano $\alpha\hat{g}$-continuous.

Theorem 3.14: The composition of two strongly nano $\alpha\hat{g}$-continuous functions is strongly nano $\alpha\hat{g}$-continuous.

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4. Perfectly Nano $\alpha\hat{g}$-continuous functions

We introduce the following definition.

Definition4.1: A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be perfectly nano $\alpha\hat{g}$-continuous if $f^{-1}(B)$ is nano clopen in U for every nano $\alpha\hat{g}$-closed set B in V.

Example 4.2: Let $U = \{a, b, c, d\}$ with $X = \{a, b, c\} \subseteq U$, $U/R = \{\{a, c\}, \{b, d\}\}$. Then $\tau_R(X) = \{U, \emptyset, \{a, c\}, \{b, d\}\}$. Also let $V = \{a, b, c, d\}$, ${}^{V}/_{R'} = \{\{a, b\}, \{c, d\}\}$, $Y = \{a, b, c\} \subseteq V$. $\tau_{R'}(Y) = \{V, \emptyset, \{a, b\}, \{c, d\}\}$. Define $f: U \to V$ by $f(a) = c, f(b) = a, f(c) = d, f(d) = b$. Here inverse image of every nano $\alpha\hat{g}$-closed sets in V is nano clopen in U. Hence f is perfectly nano $\alpha\hat{g}$-continuous.

Theorem 4.3: A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is perfectly nano $\alpha\hat{g}$-continuous if and only if $f^{-1}(B)$ is nano clopen in $(U, \tau_R(X))$ for every nano $\alpha\hat{g}$-open set B in $(V, \tau_{R'}(Y))$.

Theorem 4.4: Every perfectly nano$\alpha\hat{g}$-continuous function is strongly nano $\alpha\hat{g}$-continuous.

Proof: Follows from the definitions. **Remark 4.5** The converse of the theorem 4.4 need not be true.

Example 4.6: In example 3.2, f is strongly nano $\alpha\hat{g}$-continuous but not perfectly nano $\alpha\hat{g}$-continuous.

Theorem 4.7: If $(U, \tau_R(X))$ is extremely disconnected then every strongly nano$\alpha\hat{g}$-continuous is perfectly nano $\alpha\hat{g}$-continuous.

Proof: Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a strongly nano $\alpha\hat{g}$-continuous function. Then for every nano $\alpha\hat{g}$-closed set in V, $f^{-1}(B)$is nano closed in U. Since U is extremely disconnected, $f^{-1}(B)$ is nano clopen in U. Hence f is perfectly nano $\alpha\hat{g}$-continuous.

Theorem 4.8: Let$(U, \tau_R(X))$ be an indiscrete nano topological space, $(V, \tau_{R'}(Y))$ be a nano topological space and $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be any function, then the following statements are equivalent.

i. f is perfectly nano$\alpha\hat{g}$-continuous.
ii. f is strongly nano$\alpha\hat{g}$-continuous.

Proof: (i)⇒(ii) Follows from theorem 4.4.

(ii)⇒(i) Let B be a nano $\alpha\hat{g}$-closed set in V. By hypothesis, $f^{-1}(B)$ is nano closed in U. Since $(U, \tau_R(X))$ is an indiscrete nano topological space, $f^{-1}(B)$ is nano clopen in U. Hence f is perfectly nano $\alpha\hat{g}$-continuous.

Theorem 4.9: Every strongly nano continuous function is perfectly nano$\alpha\hat{g}$-continuous.

Proof: Follows from the definitions.

Remark 4.10: The following example shows that the converse of the above theorem need not be true.

Example 4.11: In example 4.2, f is perfectly nano $\alpha\hat{g}$-continuous, but not strongly nano continuous.

Theorem 4.12: The composition of two perfectly nano $\alpha\hat{g}$-continuous function is perfectly nano $\alpha\hat{g}$-continuous.

5. Contra Nano $\alpha\hat{g}$-continuous functions

In this section the notion of nano contra $\alpha\hat{g}$-continuity is introduced and its properties are investigated.

Definition 5.1: A function $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be contra nano $\alpha\hat{g}$-continuous if the inverse image of every nano open set in V is nano $\alpha\hat{g}$-closed in U.

Example 5.2: Let $U = \{a, b, c, d\}$ with $X = \{a, c\} \subseteq U$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$. Then the nano $\alpha\hat{g}$–closed sets are $U, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$. Also let $V =$

$\{p,q,r,s\}, {}^{V}/_{R'} = \{\{p,q\},\{r,s\}\}, Y = \{p,q\} \subseteq V$.Then $\tau_{R'}(Y) = \{V,\emptyset,\{p,q\}\}$. Define $f: U \to V$ by $f(a) = q, f(b) = r, f(c) = s, f(d) = p$. The inverse image of every nano open set in V is nano $\alpha\hat{g}$-closed in U. Hence f is contra nano $\alpha\hat{g}$-continuous.

Theorem 5.3: A function $f: (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be contra nano$\alpha\hat{g}$-continuous if and only if $f^{-1}(B)$ is nano $\alpha\hat{g}$-open in U for every nano closed set B in V.

Remark 5.4: The concept of strongly nano $\alpha\hat{g}$- continuity and contra nano $\alpha\hat{g}$-continuity are independent as shown in the following example.

Example 5.5: In example 3.2, the function *f* is strongly nano $\alpha\hat{g}$-continuous but not contra nano $\alpha\hat{g}$-continuous. In example 5.2, the function *f* is contra nano $\alpha\hat{g}$-continuous but not strongly nano $\alpha\hat{g}$-continuous.

Theorem 5.6: Every perfectly nano $\alpha\hat{g}$-continuous function is contra nano$\alpha\hat{g}$-continuous.

Proof: Follows from the definitions.

Remark 5.7: The converse of the above theorem is not true as shown in the following example.

Example 5.8: In example 5.2, *f* is contra nano$\alpha\hat{g}$-continuous function, but $f^{-1}(\{r\}) = \{b\}$ which is neither nano closed nor nano open in U. Hence *f* is not perfectly nano $\alpha\hat{g}$-continuous.

Remark 5.9: The composition of two contra nano $\alpha\hat{g}$-continuous function need not be contra nano $\alpha\hat{g}$-continuous as the following example shows.

Example 5.10: Let $U = \{a,b,c,d\}$ with $U/R = \{\{a,b\},\{c\},\{d\}\} X = \{a,c\} \subseteq U$. Then the nano $\alpha\hat{g}$-closed sets are $U,\emptyset,\{d\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$. Let $V = \{p,q,r,s\}$ with ${}^{V}/_{R'} = \{\{p,q\},\{r,s\}\}$ and $Y = \{p,q\} \subseteq V$, then $\tau_{R'}(Y) = \{V,\emptyset,\{p,q\}\}$. Also let $W = \{u,v,w,x\}$ with $W/R'' = \{\{u\},\{v,w\},\{x\}\}$ and $z = \{v,x\}$.Then $\tau_{R''}(z) = \{W,\emptyset,\{x\},\{v,w,x\},\{v,w\}\}$. Define $f: U \to V$ and $g: V \to W$ by $f(a) = q, f(b) = r, f(c) = s, f(d) = p$ and $g(p) = v, g(q) = u, g(r) = x, g(s) = w$. Here f and g are contra nano $\alpha\hat{g}$-continuous functions. But $(g \circ f)^{-1}\{x\} = f^{-1}(g^{-1}(\{x\})) = f^{-1}(\{r\}) = \{b\}$. Which is not nano $\alpha\hat{g}$-closed in U. Hence $g \circ f$ is not contra nano $\alpha\hat{g}$-continuous.

Theorem 5.11: Let $f: (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ and $g: (V,\tau_{R'}(Y)) \to (W,\tau_{R''}(Z))$ be any two functions such that $g \circ f: (U,\tau_R(X)) \to (W,\tau_{R''}(Z))$ then,

- **i.** $g \circ f$ is contra nano $\alpha\hat{g}$-continuous if g is nano continuous and f is contra nano $\alpha\hat{g}$-continuous.
- **ii.** $g \circ f$ is contra nano $\alpha\hat{g}$-continuous if g is contra nano $\alpha\hat{g}$-continuous and f is nano $\alpha\hat{g}$-irresolute.
- **iii.** $g \circ f$ is contra nano $\alpha\hat{g}$-continuous if g is contra nano $\alpha\hat{g}$-continuous and $f$is strongly nano $\alpha\hat{g}$-continuous((or) perfectly nano$\alpha\hat{g}$-continuous).

References:

[1]Bhuvaneswari.K and Mythili Gnanapriya.K.,*On nano generalized continuous functions in nano topological space*, International Journal of Mathematical Archive, 6(6)(2015),182-186.

[2]Davamani Christober.M, and Vinith Mala.A., *A Novel Concept of generalized closed sets through Nano Topology*, Infokara Research, 9(1)(2020),838-846.

[3]Lalitha.R, Francina Shalini.A., *On nano generalized$\hat{}$-continuous and irresolute functions in Nano Topological spaces*, International Journal of Engineering Science and Computing, 7(5)(2017), 11370-11374.

[4]Lellis Thivagar.M, Carmel Richard.,*On Nano forms of weakly open sets*, international Journal of Mathematics and statistics Invention.,1(1),(2013),31-37.

[5]Lellis Thivagar.M, Carmel Richard., *On Nano Continuity*, Mathematical Theory and Modelling, 3(7), (2013),32-37

[6] Lellis Thivagar.M, Saeid Jafari, Sutha Devi.V., *On new class of contra Continuity in Nano Topology*., Italian Journal of Pure and Applied Mathematics,41(2017)1-12.

[7]Levine.N., *Generalized closed sets in topology*, Rend.Circ.Math. Palermo, 19(1970), 89-96.

[8]Njastad.O.,*On some classes of nearly open sets*, Pacific Journal of Mathematics, 15(1965), 961-970.

[9]Nour.T.M, *A note on perfectly continuous functions*., European International Journal of science and technology, 8(3)(2019)(91-93).

[10]Thanga Nachiyar.R, Bhuvaneswari.K., *Nano generalized A-continuous and nano A-generalized continuous functions in nano Topological spaces*, International Journal of Mathematics Trends and Technology, 14(2)(2014), 79-83.