Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

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A Note on soft order sets in Topological Spaces

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Abstract : In this paper, we define a soft ordered topological space by adding a partial order relation to the structure of a soft topological space. Here, monotone soft sets and soft increasing (decreasing) operators are presented and their properties are analyzed in detail. We also introduce the notions of ordered soft separation axioms, namely, p-soft T_i – ordered spaces $(i = 0,1,2,3, \dots)$ with the examples. Also the equivalent conditions for p-soft regularly ordered spaces and soft normally ordered spaces are given. Moreover we define the soft ordered topological properties and verify that the property of being a p-soft T_i – ordered space is a soft ordered topological property.

Keywords: Soft increasing (decreasing) operator and p-soft T_i – ordered spaces $(i = 0,1,2,3)$.

Subject Classification : 54D10, 54D15.

1. Introduction

In 1965, Nachbin defined a topological space by adding a partial order relation to the structure of a topological space. So it can be considered that the topological ordered spaces are one of the generalizations of the topological spaces. In 1999, the notion of soft set theory was initiated by Molodstov to approach problems associated with uncertainties. He demonstrated the advantages of soft set theory compared to probability theory and fuzzy theory. The applications of soft sets in many disciplines such as Economic, Medicine, Engineering and Game theory give rise to rapidly increase researches on it. As a continuation of the study of elementary concepts regarding soft topologies, Hussain and Ahmed (2011) studied the properties of soft interior and soft boundary operators, and investigated some findings that connected between them. Recently, El Shafei et al., 2018 defined partial belong and total non-belong relations which are more effective to theoretical and application studies in soft topological spaces and then utilized them to study partial soft separation axioms. The idea of this study is to establish a soft topological ordered space which consists of a soft topological spaces endowed with a partial order relation. This paper starts by presenting the definitions and results of soft set theory and soft topological spaces which will be needed to probe results obtained herein.

Then we define the concepts of monotone soft sets and increasing (decreasing) soft operators and illuminate their fundamental properties. Also we introduce the notions of ordered soft separation axioms, namely p-soft T_i – ordered spaces $(i = 0,1,2,3, \dots)$ with the examples. Also the equivalent conditions for p-soft regularly ordered spaces and soft normally ordered spaces are given. Moreover we define the soft topological ordered properties and verify that the property of being a p-soft T_i – ordered space is a soft topological ordered property.

2. Preliminaries

Definition 2.1: (Molodstov 1999) A pair (G,A) is said to be a soft set over X provided that G is a mapping of a set of parameters A into 2^X .

* **Remark:**
  For short, G_A instead of (G,A).
  A soft set G_A can be defined as a set of ordered pairs
  $$G_E = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X \}.$$

Definition 2.2: (Maji et al. 2003) The union of soft sets G_P and F_Q over X is the soft set V_S, where $S = P \cup Q$ and a map $V : S \to 2^X$ is defined as follows

$$V(s) = \begin{cases} G(s) & : s \in P - Q \\ F(s) & : s \in Q - P \\ G(s) \cup F(s) & : s \in P \cap Q \end{cases}$$

It is written briefly, $G_P \cup F_Q = V_S$

Definition 2.3: (Pei and Miao 2005)The intersection of soft sets G_P and F_Q over X is the soft set V_S, where $S = P \cap Q$ and a map $V : S \to 2^X$ is defined by $V(s) = G(s) \cap F(s)$ for all $s \in S$. It is written briefly, $G_P \cap F_Q = V_S$

Definition 2.4: (Pei and Miao 2005)A soft set G_P is a soft subset of a soft set F_Q if X_A

i. $P \subseteq Q$
ii. For all $p \in P$, $G(p) \subseteq F(p)$.

The soft sets G_P and F_Q are soft equal if each of them is a soft subset of the other. The set of all soft sets, over X under a parameter set P, is denoted by S (X_A).

It should be noted that there are other kinds of soft subset and soft equal relations were introduced and discussed in Qin and Hong (2010)**.**

Definition 2.5: (Ali et al. 2009)The relative complement of a soft set G_A, denoted by $G_A{}^C$, where $G^C: A \to 2^X$ is the mapping defined by $G^C(a) = X \setminus G(a)$, for each $a \in A$.

Definition 2.6: (Shabir and Naz 2011)A collection τ of soft sets over X under a fixed parameters set A is said to be a soft topology on X if it satisfies the following three axioms:

i. X and $\emptyset$ belong to τ.
ii. The intersection of a finite family of soft sets in τ belongs to τ.
iii. The union of arbitrary family of soft sets in τ belongs to τ.

The triple (X, τ, A) is called a soft topological space(STS). Every member of τ is called soft open and its relative complement is called soft closed.

Definition 2.7: (Shabir and Naz 2011) A soft set x_A over X is defined by $x(a) = \{x\}$, for each $e \in E$.

Definition 2.8: (Shabir and Naz 2011)For a soft subset H_A of an STS (X, τ, A), *Int* (H_A) is the largest soft open set contained in H_A and *Cl*(H_A) is the smallest soft closed set containing H_A.

Definition 2.9: (Zorlutuna et al. 2012)A soft subset W_A of an STS (X, τ, A) is called soft neighborhood of x ∈ X, if there exists a soft open set G_A such that x ∈ $G_A \subseteq W_A$.

Definition 2.10: (Zorlutuna et al. 2012)A soft mapping between S(X_A) and S (Y_B) is a pair (g, φ) denoted also by g_φ, of mappings such that g: $X \rightarrow Y$, φ: $A \rightarrow B$. Let G_M and F_L be soft subsets of S(X_A) and S (Y_B), respectively. Then the image of G_M and pre−image of F_L are defined by

i. g_φ (G_M) = $\left(g_\varphi(G)\right)_B$ is a soft subset of S (Y_B) such that

$$g_\varphi(G)(b)=\begin{cases} \bigcup_{a\in\varphi^{-1}(b)\cap k} g\left(G(a)\right) & : \varphi^{-1}(b)\cap k \neq \emptyset \\ \emptyset & : \varphi^{-1}(b)\cap k = \emptyset \end{cases}$$

for each b ∈ B.

ii. ${g_\varphi}^{-1}$ (F_L) = $\left({g_\varphi}^{-1}(F)\right)_A$ is a soft subset of S(X_A) such that

$$ {g_\varphi}^{-1}(F)(a)=\begin{cases} g^{-1}\left(F\left(\varphi(a)\right)\right) & : \varphi(a) \in L \\ \emptyset & : \text{otherwise} \end{cases}$$

for each a ∈ A

.

Definition 2.11: (El Shafei et al. 2018)An STS (X, τ, A) is said to be :

i. p−soft T_0 space if for every pair of distinct points $x, y \in X$, there is a soft open set G_A such that $x \in G_A, y \notin G_A$ or $y \in G_A, x \notin G_A$.
ii. p−soft T_1 space if for every pair of distinct points $x, y \in X$, there are soft open sets G_A and F_A such that $x \in G_A, y \notin G_A$ and $y \in F_A, x \notin F_A$.
iii. p−soft T_2 space if for every pair of distinct points $x, y \in X$, there are disjoint soft open sets G_A and F_A containing x and y respectively.
iv. p−soft regular if for every closed set H_A and $x \in$ X, such that $x \notin H_A$, there are disjoint soft open sets G_A and F_A such that $H_A \subseteq G_A$ and $x \in F_A$.
v. (Shabir and Naz 2011) Soft normal if for every two disjoint soft closed sets H_{1A} and H_{2A} , there are two disjoint soft open sets G_A and F_A such that $H_{1A} \subseteq G_{1A}$ and $H_{2A} \subseteq F_E$.
vi. p−soft T_3 space if it is both p−soft regular and p−soft T_1 space.
vii. p−soft T_4 space if it is both soft normal and p−soft T_1 space.

Definition 2.12: (Kelly 1975)A binary relation ≤ on a non empty set X is called a partial order relation if it is reflexive, anti-symmetric and transitive.

Definition 2.13: (Kelly 1975)Let (X, ≤) be a partially ordered set. An element a in X is

i. A smallest element of X provided that $a \leq x$, for each $x \in X$.

ii. A largest element of X provided that $x \leq a$, for each $x \in X$.

Definition 2.14: (Nachin 1965) A triple (X, τ, $\leq$) is said to be topological ordered space, where (X, $\leq$) is a partially ordered set and (X, τ) is a topological space.

Definition 2.15: (McCartan 1968) A topological ordered space (X, τ, $\leq$) is called

i. Lower (Upper) T_1 ordered if for each $x \nleq y$ in X, there is an increasing (decreasing) neighborhood W of a such that b belongs to W^c

ii. T_0 ordered if it is lower T_1 ordered or upper T_1 ordered .

iii. T_1 ordered if it is lower T_1 ordered and upper T_1 ordered .

iv. T_2 ordered if for each $x \nleq y$ in X, there are disjoint neighborhoods W_1 and W_2 of x and y, respectively , such that W_1 is increasing and w_2 is decreasing.

3. Soft monotone sets:

In this section, we first formulate the definitions of partially soft ordered sets, increasing (decreasing) soft sets and increasing (decreasing, ordered embedding) soft maps. Then we move on to the main properties of these new concepts.

Definition 3.1 : Let $\leq$ be a partial order relation on a non-empty set and let A be a set of parameters. A triple (X, A , $\leq$) is said to be a partially soft ordered set.

Definition 3.2 : Let (X, A , $\leq$) be a partially soft ordered set. We define a soft increasing operator ρ : $(S(X_A), \leq) \rightarrow (S(X_A), \leq)$ and a soft decreasing operator $\sigma :(S(X_A), \leq) \rightarrow (S(X_A), \leq)$ as follows, for each soft subset G_A of $S(X_A)$

i. $\rho(G_A) = (\rho G)_A$, where ρG is a mapping of A into X given by ρ G(a) $=\rho\left(G(a)\right) =\{x \in X : y \leq x \; for \; some \; y \in G(a)\}$.

ii. $\sigma(G_A) = (\sigma G)_A$, where σG is a mapping of A into X given by σG(a) $=\sigma\left(G(a)\right) =\{x \in X : x \leq y \; for \; some \; y \in G(a)\}$.

Definition 3.3 : A soft subset G_A of a partially soft ordered set (X, A , $\leq$) is said to be:

i. Increasing if $G_A = \rho(G_A)$.

ii. Decreasing if $G_A = \sigma(G_A)$.

Proposition 3.4 : We have the following results for a soft subset G_A of a partially soft ordered set (X, A , $\leq$).

i. G_A is increasing if and only if for each $E_a^y \in \rho(G_A)$, then $E_a^y \in G_A$.

ii. G_A is decreasing if and only if for each $E_a^y \in \sigma(G_A)$, then $E_a^y \in G_A$.

iii. If G_A is increasing , then for each $x \in \rho(G_A)$, we have $x \in G_A$.

iv. If G_A is decreasing , then for each $x \in \sigma(G_A)$, we have $x \in G_A$.

Proof: Case (i) only prove, and the other follows the same.

Necessary : It easily followed from Definition 3.3.

Sufficient : By hypothesis, $E_a^y \in \rho(G_A)$ implies that G_A. Then x $\in G(a)$. Since $\leq$ is reflexive, then x $\in \rho(G_A)$. So $E_a^y \in \rho(G_A)$. This means that $\rho(G_A) \subseteq G_A$. Thus $G_A = \rho(G_A)$. Hence a soft set G_A *is increasing*.

Theorem 3.5: The finite product of soft increasing (decreasing) sets is increasing (decreasing) .

Proof: We only prove the theorem for two soft sets in case of soft increasing sets and one can prove it similarly for finite soft sets.

Let G_M and F_N be two increasing soft subsets of (X, M, $\leq_1$) and (Y, N, $\leq_2$) respectively. Setting $H_{MXN} = G_M$ x F_N such that $H(m,n) = G(m)\, X\, F(n)$, for each $(m,n) \in M\, X\, N$. Suppose , to the contrary, H_{MXN} is not increasing.

Then there exists a soft point $C_{(\gamma,\delta)}^{(x,y)}$ such that $C_{(\gamma,\delta)}^{(x,y)} \in \rho H_{MXN}$ and $C_{(\gamma,\delta)}^{(x,y)} \notin H_{MXN}$. This means that $(x,y) \in \rho H(\gamma,\delta)$ and $(x,y) \notin H(\gamma,\delta)$. So $(x,y) \in iG(\gamma)\, X\, F(\delta)$ implies that

$x \in iG(\gamma) = G(\gamma)$ and $y \in \rho F(\delta) = F(\delta)$ (1)

and $(x,y) \notin G(\gamma)\, X\, F(\delta)$ implies that

$x \notin G(\gamma)$ or $y \notin F(\delta)$ (2)

From (1) and (2) we got a contradiction. Since the contradiction arises by assuming that the soft set H_{MXN} is not increasing , then H_{MXN} is increasing.

4. Soft ordered separation axioms:

In this section , we introduce the soft ordered separation axioms namely , p-soft T_i – ordered spaces $(i = 0,1,2,3, \ldots)$ and to studying their properties.

Definition 4.1: A system (X, τ, A, $\leq$) is said to be a soft ordered topological space, where (X, τ, A) is a soft topological space and (X, τ, $\leq$) is a partially soft ordered set.

Definition 4.2: A soft ordered topological space (X, τ, A, $\leq$) is said to be :

i. Lower (Upper) p-soft T_1 ordered if for every distinct points x $\nleq$ y in X, there exists an soft increasing (decreasing) neighborhood W_A of x such that $y \nleq W_A$.

ii. p-soft T_0 ordered if it is soft lower T_1 ordered or soft upper T_1 ordered .

iii. p $-$ soft T_1 ordered if it is soft lower T_1 ordered and soft upper T_1 ordered .

iv. p-soft T_2 ordered if for every distinct points x $\nleq$ y in X, there exists disjoint soft neighborhoods W_A and V_A of x and y, respectively , such that W_A is increasing and V_A is decreasing.

Proposition 4.3 : Every p $-$ soft T_i ordered space (X, τ, A, $\leq$) is p $-$ soft T_{i-1} ordered space for $i = 1,2$.

Proof: It is immediately followed from the above definition.

Example 4.4: Let A= $\{a_1, a_2\}$ be a set of parameters, $\leq = \Delta \cup \{(1,y): y \in R\}$ be a partial order relation on the set of real numbers R and $\tau = \{\emptyset, G_A \subseteq R: G_A^C\ is\ finite\}$ be a soft topology on R. Trivially, $(R, \tau, A, \leq)$ is p $-$ soft T_1 ordered but not p $-$ soft T_2 ordered space.

Theorem 4.5: Every p − soft T_i ordered space (X, τ, A, $\leq$) is p − soft T_i ordered space for $i = 0,1,2$.
Proof: The proof follows from the definition of p − soft T_i ordered spaces and the definition of p − soft T_i spaces, for $i = 0,1,2$.

Conclusion: In this paper, we define a soft ordered topological space by adding a partial order relation to the structure of a soft topological space. Some concepts of monotone soft sets and soft increasing (decreasing) operators are presented with their properties. We also introduce the notions of ordered soft separation axioms, namely, p-soft T_i – ordered spaces $(i = 0,1,2,3, \ldots)$ with the examples. In future, we plan to introduce and study new soft ordered separation axioms by using different operators.

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