Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

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Dr. T. Tamizh Chelvam
Dr. T. Asir

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A NOTE ON COMPLEMENT OF THE REDUCED NON-ZERO COMPONENT GRAPH OF FREE SEMI-MODULES

K. PRABHA ANANTHI [1] AND T. TAMIZH CHELVAM[2]

Abstract. In this paper, we discuss about certain graphs from vector spaces and graph from semimodules. More specifically, we present about the complement of the reduced non-zero component graph($\overline{\Gamma^*}(\mathbb{M})$) and we show that the graph $\overline{\Gamma^*}(\mathbb{M})$is connected and find its girth. Also, we give a necessary and sufficient condition for $\overline{\Gamma^*}(\mathbb{M})$ to be complete and complete bipartite.

1. Introduction

The interdisciplinary study of graphs from algebraic structures is an optimal trend in research area nowadays. With respect to the study of graphs associated with various algebraic structures, enormous number of research publications ensures its utility from the idea of zero divisor graph of a commutative ring with unity. The concept of graph from a commutative ring was introduced by Beck [3] and later modified and named as zero-divisor graphs by Anderson and Livingston [2]. Till now, lot of researchers [1, 2] have worked on graph structures from various algebraic structures. In this direction, Das [7, 8] has introduced and investigated a graph called the non-zero component graph of a finite dimensional vector space. Recently, it was generalized for semimodules by Bhuniya and Maity [5] and named the graph as the reduced non-zero component graph $\Gamma^*(\mathbb{V})$. In this paper, we study about the complement of the reduced non-zero component graph [7, 5] of a finitely generated free semimodule $\mathbb{M}$ over a semiring $\mathbb{S}$ with identity having invariant free basis number property.

2. Preliminaries

In this section, we recall certain notation, concepts, and results concerning elementary graph theory which will be needed in the subsequent sections. By a graph $G = (V, E)$, we mean an undirected simple graph with vertex set V and edge set E. The *complement* $\overline{G}$ of graph G is the graph whose vertex set is $V(G)$ and two vertices u, v are adjacent in $\overline{G}$ if and only if they are not adjacent in G. For a subset $A \subseteq V(G)$, $\langle A \rangle$ denotes the induced subgraph of G. Two graphs $G = (V, E)$ and $G' = (V', E')$ are said to be *isomorphic* if there exists a bijection $\varphi : V \longrightarrow V'$ such that $(u, v) \in E$ if and only if $(\varphi(u), \varphi(v)) \in E'$. The *diameter* of a graph is defined as $diam(G) = \max_{u,v \in V} d(u, v)$, the largest distance between pairs of vertices of the graph, if it exists. Otherwise, $diam(G)$ is defined as ∞. For unspecified terms in graph theory, one may refer to[6, 14] and we refer to Golan[10] for basic notions and results on semirings and semimodules.

Let us recall the definition of non-zero component graph of a finite dimensional vector space and some results from [7, 8, 12].

Definition 2.1. ([7], Graph from Vector Spaces) *Let $\mathbb{V}$ be a vector space of dimension k over a field $\mathbb{F}$. Assume that $\{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ is a basis and θ is the null vector. Then, any vector* $\mathbf{a} \in \mathbb{V}$ *can be expressed uniquely as a linear combination of the form* $\mathbf{a} = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_k\alpha_k$. *This representation of* $\mathbf{a}$ *is called as its basic representation with respect to* $\{\alpha_1, \alpha_2, \ldots, \alpha_k\}$. *The non-zero component graph of* $\mathbb{V}$ [7], *denoted by* $\Gamma(\mathbb{V})$, *is a simple undirected graph with vertex set*

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as non-zero vectors of $\mathbb{V}$ *and such that there is an edge between two distinct vertices* a, b *if and only if there exists at least one* α_i *along which both* a *and* b *have non-zero scalars.*

The following are the results proved in [7, 8, 12].

Theorem 2.2. ([7, Theorem 4.1]) *Let* $\mathbb{V}$ *be a vector space of dimension* k *over a field* $\mathbb{F}$*. Then the non-zero component graph* $\Gamma(\mathbb{V})$ *is connected and diam(*$\Gamma(\mathbb{V})$*)=2.*

Theorem 2.3. ([7, Theorem 4.2]) *Let* $\mathbb{V}$ *be a vector space of dimension* k *over a field* $\mathbb{F}$*. Then the non-zero component graph* $\Gamma(\mathbb{V})$ *is complete if and only if* $\mathbb{V}$ *is one-dimensional.*

Theorem 2.4. ([7, Theorem 7.1]) *Let* $\mathbb{V}$ *be a vector space over a finite field* $\mathbb{F}$ *with* q *elements and* Γ *be its associated graph with respect to a basis* $\{\alpha_1, \alpha_2, \ldots, \alpha_k\}$*. Then, the degree of the vertex* $c_1\alpha_{i_1} + c_2\alpha_{i_2} + \cdots + c_s\alpha_{i_s}$ *in* $\Gamma(\mathbb{V})$*, where* $c_1c_2\ldots c_s \neq 0$*, is* $(q^s-1)q^{k-s}-1$*.*

Lemma 2.5. ([8, Lemma 3.1]) *If* $\mathbb{V}$ *be a* k*-dimensional vector space over a finite field* $\mathbb{F}$ *with* q *elements, then the minimum degree* δ *of* $\Gamma(\mathbb{V})$ *is* $q^{k-1}(q-1)-1$*.*

Theorem 2.6. ([8, Theorem 3.6]) *If* $\mathbb{V}$ *be a* k*-dimensional vector space over a finite field* $\mathbb{F}$ *with* q *elements, then the order* n *of* $\Gamma(\mathbb{V})$ *is* q^k-1 *and the size* m *of* $\Gamma(\mathbb{V})$ *is* $\frac{q^{2k}-q^k+1-(2q-1)^k}{2}$*.*

One of the most important topological properties of a graph is its genus. In the paper [12], we characterized the planar and toroidal nature of $\Gamma(\mathbb{V})$. More specifically, we characterized all finite dimensional vectors spaces $\mathbb{V}$ over finite fields for which $\Gamma(\mathbb{V})$ has genus either 0 or 1 or 2. The results in that respect are given below.

Theorem 2.7. ([12, Theorem 4.1]) *Let* $k \geq 1$ *and* $q \geq 2$ *be integers. Let* $\mathbb{V} = \mathbb{F}^{(k)}(\mathbb{F})$ *be a* k*-dimensional vector space over a field* $\mathbb{F}$ *with* q *elements. Then* $\Gamma(\mathbb{V})$ *is planar if and only if either* $k = 1$ *and* $q \leq 5$ *(or)* $k = 2$ *and* $q = 2$ *(or)* $k = 3$ *and* $q = 2$*.*

Theorem 2.8. ([12, Theorem 4.3]) *Let* $k \geq 1$ *and* $q \geq 2$ *be integers. Let* $\mathbb{V} = \mathbb{F}^{(k)}(\mathbb{F})$ *be a* k*-dimensional vector space over a field* $\mathbb{F}$ *with* q *elements. Then* $\Gamma(\mathbb{V})$ *is toroidal if and only if* $k = 1$ *and* $q = 7, 8$*.*

Theorem 2.9. ([12, Theorem 4.5]) *Let* $\mathbb{V} = \mathbb{F}^{(k)}(\mathbb{F})$ *be a vector space over a field* $\mathbb{F}$ *with* q *elements and* $k \geq 1$*,* $q \geq 2$*. Then* $g(\Gamma(\mathbb{V})) = 2$ *if and only if either* $k = 1$ *and* $q = 9$ *or* $k = 2$ *and* $q = 3$*.*

Recently, the non-zero component graph of a finite dimensional vector space was generalized for semimodules over semiring with identity by Bhuniya and Maity [5] and named the graph as the reduced non-zero component graph $\Gamma^*(\mathbb{V})$. Let us recall some basic definitions related to semiring and semimodules and some results in [5].

Definition 2.10. *A semimodule over a semiring* $\mathbb{S}$ *is a commutative monoid* $(\mathbb{M}, +)$ *with identity* θ *for which we have a function* $\mathbb{S}\times\,\mathbb{M} \longrightarrow \mathbb{M}$*, denoted by* $(c, x) \longrightarrow cx$ *and called scalar multiplication, which satisfies the following conditions: for all* $c, d \in \mathbb{S}$ *and* $x, y \in \mathbb{M}$ *:*
$(i)(c+d)x = cx + dx;$
$(ii)c(x+y) = cx + cy;$
$(iii)(cd)x = c(dx);$
$(iv)1x = x;$
$(v)c\theta = \theta = 0x.$

Let B be a non-empty subset of $\mathbb{M}$. Then we denote $span(B) = \{\sum_{i=1}^{n} c_ix_i : n \in \mathbb{N}, c_i \in \mathbb{S}, x_i \in B\}$. If $span(B) = \mathbb{M}$, then B is called a generating subset of $\mathbb{M}$. A semimodule $\mathbb{M}$ having a finite generating set B is called finitely generated. A nonempty subset D of vectors in $\mathbb{M}$ is called linearly dependent if there exists $x \in D$ such that $x \in span(D \setminus \{x\})$; Otherwise it is called linearly independent and free if every element of $\mathbb{M}$ is expressed as a linear combination of elements of D in at most one way. It is easy to see that every free subset of $\mathbb{M}$ is linearly independent. A linearly independent generating subset of $\mathbb{M}$ is called a basis of $\mathbb{M}$ and a free generating subset of $\mathbb{M}$ is called a free basis of $\mathbb{M}$ [13]. If $\mathbb{M}$ has a free basis then it is called a free semimodule. It is easy to see that every finitely generated semimodule has a basis [13]. Also every free basis is a basis.

Definition 2.11. *A semiring $\mathbb{S}$ is said to have invariant free basis number property if any two bases of a finitely generated free semimodule over $\mathbb{S}$ have the same cardinality.*

If $\mathbb{S}$ is a semiring having invariant free basis number property, then from Corollary 3.1 [13], it follows that every vector of a finitely generated free semimodule $\mathbb{M}$ over $\mathbb{S}$ is expressed uniquely in terms of each basis. The cardinality of a basis of $\mathbb{M}$ is denoted by $dim(\mathbb{M})$. Henceforth, unless stated otherwise, $\mathbb{S}$ is a semiring having invariant free basis number property and $\mathbb{M}$ is a finitely generated free semimodule over $\mathbb{S}$. Isomorphism between two semimodules is defined as in usual linear algebra. It follows from Corollary 5.2 [11], that two semimodules $\mathbb{M}_1$ and $\mathbb{M}_2$ are isomorphic if and only if $dim(\mathbb{M}_1) = dim(\mathbb{M}_2)$.

If $X = \{x_1, x_2, \ldots, x_k\}$ is a basis of a semimodule $\mathbb{M}$, then every vector $v \in \mathbb{M}$ can be expressed uniquely as $v = c_1x_1 + \cdots + c_kx_k; c_i \in \mathbb{S}$. We call c_i the i^{th} component of v and is denoted by v_i.

Definition 2.12. ([5], Graph from semimodules) *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$ with identity having invariant free basis number property and $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ is a basis for $\mathbb{M}$. Then the reduced non-zero component graph $\Gamma^*(\mathbb{M})$ [5] of $\mathbb{M}$ with respect to the basis α, is the graph with vertex set $V = \mathbb{M}^* \setminus \{\sum\limits_{i=1}^{k} c_i\alpha_i : c_i \neq 0 \ \forall \ i\}$ and two distinct vertices $a, b \in V$ are adjacent if there exists i such that both a_i, b_i are non-zero.*

The following are the results proved in [5].

Theorem 2.13. ([5, Theorem 3.1]) *If $dim(\mathbb{M}) = k \geq 3$, $\Gamma^*(\mathbb{M})$ is connected and $diam(\Gamma^*(\mathbb{M})) = 2$.*

Theorem 2.14. ([5, Theorem 3.2]) *The domination number of $\Gamma^*(\mathbb{M})$ is 2.*

Theorem 2.15. ([5, Theorem 3.3]) *If $D = \{y_1, y_2, \ldots, y_l\}$ is a minimal dominating set of $\Gamma^*(\mathbb{M})$, then $l \leq k$.*

Theorem 2.16. ([5, Theorem 3.4]) *The independence number of $\Gamma^*(\mathbb{M})$ is $dim(\mathbb{M}) = k$.*

In this paper, we study about the complement of the reduced non-zero component graph [7, 5] of a finitely generated free semimodule $\mathbb{M}$ over a semiring $\mathbb{S}$ with identity having invariant free basis number property. The following are the definition of complement graph of $\Gamma^*(\mathbb{M})$.

Definition 2.17. *The complement graph of $\Gamma^*(\mathbb{M})$ is defined as the graph $\overline{\Gamma^*}(\mathbb{M}_\alpha) = (V, E)$ (or simply $\overline{\Gamma^*}(\mathbb{M})$) with respect to α as follows: $V = \mathbb{M}^* \setminus \{\sum\limits_{i=1}^{k} c_i\alpha_i : c_i \neq 0 \ \forall \ i\}$ and for $a, b \in V$, $a \sim b$ or $(a, b) \in E$ if and only if there exists no i such that both a_i, b_i are non-zero. Unless otherwise mentioned, we take the basis on which the graph is constructed as $\{\alpha_1, \alpha_2, \ldots, \alpha_k\}$.*

Throughout this paper, we take the semimodules $\mathbb{M}$ are finitely generated and free over a semiring $\mathbb{S}$ with identity having invariant free basis number property and with $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ as a basis and $k = \dim_{\mathbb{S}}(\mathbb{M})$ or $(rank_{\mathbb{S}}(\mathbb{M}))$.

3. Basic properties of $\overline{\Gamma^*}(\mathbb{M})$

The following concerns about $\overline{\Gamma^*}(\mathbb{M}_\alpha)$ and $\overline{\Gamma^*}(\mathbb{M}_\beta)$ with respect to two bases α and β of M of equal cardinality.

Theorem 3.1. *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$ with two bases $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ and $\beta = \{\beta_1, \beta_2, \ldots, \beta_k\}$ of $\mathbb{M}$. Then the graphs $\overline{\Gamma^*}(\mathbb{M}_\alpha)$ and $\overline{\Gamma^*}(\mathbb{M}_\beta)$ are isomorphic.*

Proof. Define $\Phi : \mathbb{M} \longrightarrow \mathbb{M}$ by $\Phi(c_1\alpha_1 + c_2\alpha_2 + \cdots + c_k\alpha_k) = c_1\beta_1 + c_2\beta_2 + \cdots + c_k\beta_k$. Clearly Φ is an $\mathbb{S}$-semimodule isomorphism on $\mathbb{M}$ such that $\Phi(\alpha_i) = \beta_i$ for all $i \in \{1, 2, \ldots, k\}$. It is enough to show that the restriction of Φ on non-null vectors of $\mathbb{M}$ such that at least one scalar is zero induces a graph isomorphism $\Phi' : \overline{\Gamma^*}(\mathbb{M}_\alpha) \longrightarrow \overline{\Gamma^*}(\mathbb{M}_\beta)$. Clearly, Φ' is a bijection. Let $a = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_k\alpha_k$ and $b = b_1\alpha_1 + b_2\alpha_2 + \cdots + b_k\alpha_k$ with a is not adjacent to b in $\overline{\Gamma^*}(\mathbb{M}_\alpha)$. Then, $\exists\ i \in \{1, 2, \dots, k\}$ such that both a_i and b_i are non-zero. Hence $\Phi'(a)$ is not adjacent to $\Phi'(b)$ in $\overline{\Gamma^*}(\mathbb{M}_\beta)$. Similarly, it can be shown that if a and b are adjacent in $\overline{\Gamma^*}(\mathbb{M}_\alpha)$, then $\Phi'(a) \sim \Phi'(b)$ in $\overline{\Gamma^*}(\mathbb{M}_\beta)$. □

Now we shall investigate some of the basic properties such as connectedness, diameter of $\overline{\Gamma^*}(\mathbb{M})$. The following are trivial.

Note 3.2. If $\dim(\mathbb{M}) = 1$, then $\overline{\Gamma^*}(\mathbb{M})$ is null graph. Further if $\dim(\mathbb{M}) = 2$ and $|\mathbb{S}| = 2$, then $\overline{\Gamma^*}(\mathbb{M})$ is K_2.

In the following results, we prove that $\overline{\Gamma^*}(\mathbb{M})$ is connected in the remaining cases.

Lemma 3.3. *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$. If $\dim(\mathbb{M}) = 2$ and $|\mathbb{S}| \geq 3$, then $\overline{\Gamma^*}(\mathbb{M})$ is connected and $diam(\overline{\Gamma^*}(\mathbb{M})) = 2$.*

Proof. Let a and b be two distinct vertices in $\overline{\Gamma^*}(\mathbb{M})$. If a and b are adjacent in $\overline{\Gamma^*}(\mathbb{M})$, then $d(a, b) = 1$. Suppose a and b are not adjacent. Since $\dim(\mathbb{M}) = 2$, either of the following is true for scalars of a and b:
(i) $a_1 \neq 0$, $a_2 = 0$, $b_1 \neq 0$ and $b_2 = 0$;
(ii)$a_1 = 0$, $a_2 \neq 0$, $b_1 = 0$ and $b_2 \neq 0$;
When (i) is true. Since $|\mathbb{S}| \geq 3$, there exists a vertex $c \in \overline{\Gamma^*}(\mathbb{M})$ with $c_1 = 0$ and $c_2 \neq 0$. Then, $a - c - b$ is a path in $\overline{\Gamma^*}(\mathbb{M})$. Therefore $d(a, b) = 2$. Similar fact is true in the case of (ii). □

Theorem 3.4. *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$. If $\dim(\mathbb{M}) \geq 3$, then $\overline{\Gamma^*}(\mathbb{M})$ is connected and $diam(\overline{\Gamma^*}(\mathbb{M})) = 3$.*

Proof. Let $\dim(\mathbb{M}) = k$. Let a and b be two distinct vertices in $\overline{\Gamma^*}(\mathbb{M})$. If a and b are adjacent in $\overline{\Gamma^*}(\mathbb{M})$, then $d(a, b) = 1$. Otherwise, there exists at least one $i \in \{1, 2, \dots k\}$ such that $a_i, b_i \neq 0$. Since $a, b \in V(\overline{\Gamma^*}(\mathbb{M}))$, there exists j and ℓ in $\{1, 2, \dots k\}$ such that a_j and b_ℓ are zero.

If $j = \ell$, then we take a vertex c such that $c_j \neq 0$ and $c_i = 0$ for all $1 \leq i \neq j \leq k$. Then $a - c - b$ is a path in $\overline{\Gamma^*}(\mathbb{M})$ and hence $diam(\overline{\Gamma^*}(\mathbb{M})) = 2$.

If $j \neq \ell$, then we take a vertex d with $d_\ell \neq 0$ and $d_i = 0$ for all $1 \leq i \neq \ell \leq k$. Hence $a - c - d - b$ is a path in $\overline{\Gamma^*}(\mathbb{M})$ and so $diam(\overline{\Gamma^*}(\mathbb{M})) = 3$. □

Now we characterize when $\overline{\Gamma^*}(\mathbb{M})$ is complete bipartite.

Theorem 3.5. *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$ of dimension k. Then $\overline{\Gamma^*}(\mathbb{M})$ is complete bipartite if and only if $\dim(\mathbb{M}) = 2$.*

Proof. Assume that $\overline{\Gamma^*}(\mathbb{M})$ be complete bipartite. Suppose $\dim(\mathbb{M}) > 2$. Then there exist $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{M}$ such that $\alpha_1, \alpha_2, \alpha_3$ are in a basis of $\mathbb{M}$. It is easy to observe that the induced subgraph $\langle\{\alpha_1, \alpha_2, \alpha_3\}\rangle$ of $\overline{\Gamma^*}(\mathbb{M})$ is K_3, which is a contradiction to $\overline{\Gamma^*}(\mathbb{M})$ does not contain an odd cycle as a bipartite graph. Therefore $\dim(\mathbb{M}) = 2$.

Conversely, assume that $\dim(\mathbb{M}) = 2$ and $\{\alpha_1, \alpha_2\}$ is a basis of $\mathbb{M}$. Since $\dim(\mathbb{M}) = 2$, the vertices of $\overline{\Gamma^*}(\mathbb{M})$ is of the form $\{(a, b) \in \mathbb{M} : a = 0 \text{ or } b = 0\}$. Note that $H_1 = \{(a, 0) \in \mathbb{M} : a \neq 0\}$ and $H_2 = \{(0, b) \in \mathbb{M} : b \neq 0\}$ are independent sets and every vertex in H_1 is adjacent to every vertex in H_2. □

Next, we obtain the girth of $\overline{\Gamma^*}(\mathbb{M})$.

Theorem 3.6. *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$ of dimension k. Then*

$$gr(\overline{\Gamma^*}(\mathbb{M})) = \begin{cases} 3 \ if \ k \geq 3; \\ 4 \ if \ k = 2 \ and \ |\mathbb{S}| \geq 3; \\ \infty \ if \ k = 2 \ and \ |\mathbb{S}| = 2. \end{cases}$$

Proof. **Case 1.** Let $k \geq 3$. Assume that A be a basis for $\mathbb{M}$ with $|A| = k \geq 3$. The subgraph induced by A is complete and $\langle A \rangle$ is K_k. Hence $gr(\overline{\Gamma^*}(\mathbb{M})) = 3$.

Case 2. Let $k = 2$ and $|\mathbb{S}| \geq 3$. By Theorem 3.5, $\overline{\Gamma^*}(\mathbb{M})$ is complete bipartite and hence $gr(\overline{\Gamma^*}(\mathbb{M})) = 4$.

Case 3. If $k = 2$ and $|\mathbb{S}| = 2$, then by Note 3.2, $\overline{\Gamma^*}(\mathbb{M})$ is K_2 and hence $gr(\overline{\Gamma^*}(\mathbb{M})) = \infty$. $\square$

Now we characterize when $\overline{\Gamma^*}(\mathbb{M})$ is complete.

Theorem 3.7. *Let $\mathbb{M}$ be a finitely generated free semimodule over a semiring $\mathbb{S}$. Then $\overline{\Gamma^*}(\mathbb{M})$ is complete if and only if* $\dim(\mathbb{M}) = 2$ *and* $|\mathbb{S}| = 2$.

Proof. Assume that $\overline{\Gamma^*}(\mathbb{M})$ is complete. Suppose $\dim(\mathbb{M}) \geq 3$. By [5, Theorem 3.1], $\Gamma^*(\mathbb{M})$ is connected and $diam(\Gamma^*(\mathbb{M})) = 2$, which is a contradiction. Therefore $\dim(\mathbb{M}) \leq 2$. By Note 3.2, we have $\dim(\mathbb{M}) = 2$. Suppose $|\mathbb{S}| \geq 3$, then (1,0) is not adjacent to $(a_1, 0)$ which is a contradiction. Hence $|\mathbb{S}| = 2$.

Conversely, assume that $\dim(\mathbb{M}) = 2$ and $|\mathbb{S}| = 2$. Then $\overline{\Gamma^*}(\mathbb{M})$ is K_2 as seen in Note 3.2. $\square$

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Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli 627 012, Tamil Nadu, India[1,2]

E-mail address: ksprabhaananthi@gmail.com[1]; tamche59@gmail.com[2]