Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

This volume makes available a record of articles presented in the conference. This volume contains the papers presented in the conference without any referring process.

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Power 3 Mean Cordial Labeling of Graphs

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Abstract:

Let f be a function from $V(G)$ to $\{0,1,2,3,4\}$. For each edge uv of G, assign the label $f(e=uv) = \left\lceil \left(\frac{f(u^3)+f((v^3)}{2}\right)^{\frac{1}{3}} \right\rceil$ f is called a Power 3 Mean Cordial labeling of G, if $|V_f$ (i)$-V_f$(j) ≤1 and $|e_f$(i)-e_f(j)| ≤1,i,j ∈ {0,1,2,3,4} where V_f(x) and e_f(x) denote the number of vertices and edges labeled with $x(x = 0,1,2,3,4)$ respectively. A graph with a Power 3 mean cordial labeling is called **a Power 3 mean cordial graph.**

Mathematics subject classification:05C78

Keywords : Graph, Power 3 Mean Graph, Path, Cycle, Comb, Power 3Mean Graphs ,Power 3 Mean Cordial graphs

1.Introduction:

We begin with simple, connected, undirected graph $G = V(G), E(G)$ without loops or parallel edges. For a detailed survey of labeling, we refer to J.A. Gallian[4]. For all other standard terminology and notations we follow[3]. The concept of Mean Cordial labeling was introduced in [1]. Motivated by above results and by the motivation of the authors we study power 3 Mean labeling was introduced in [5].

A PathP_n is a walk in which all the vertices are distinct.The graph obtained by joining a single pendent edge to each vertex of a path is called comb. $P_n \odot K_{1,2}$ is a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$.

Definition 1.1

A graph G with p vertices and q edges is called a power -3 mean graph,if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2,\dots\dots,q+1$ in such a way that in each edge $e = uv$ is labelled with $f(e = uv) = \left\lceil \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rceil$ or $\left\lfloor \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rfloor$. Then , the edge labels are distinct. In this case f is called Power 3 Mean labelling of G.

Definition 1.2

Let f be a function from $V(G)$ to $\{0,1,2,3,4\}$. For each edge $u\,v$ of G, assign the label $f(e = uv) =$ f is called a Power 3 Mean Cordial labeling of G, if $|V_f(i) - V_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0,1,2,3,4\}$ where $V_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x(x = 0,1,2,3,4)$ respectively. A graph with a Power 3 Mean Cordial labeling is called **a Power 3 Mean Cordial graph** .

Theorem 1.3: Any Path P_n is a power 3 mean graph.

Theorem 1.4: Any Comb $P_{n \odot K_1}$ is a Power 3 mean graph.

Theorem 1.5: $P_n \odot K_{1,2}$ is a Power 3 mean graph.

2.Main Results

Theorem :2.1

Path P_n is a Power 3 Mean cordial graph

Proof:

Let P_n be the path on n vertices $u_1, u_2, \dots\dots. u_n$

Case (i): $n \equiv 0(mod5)$

Let $n = 5t$

We define the function $f: V(P_n) \to \{0,1,2,3,4\}$ by

$f(u_i) = 4 \ \ 1 \leq i \leq t$; $f(u_{t+i}) = 3 \ \ 1 \leq i \leq t$

$f(u_{2t+i}) = 2 \ \ 1 \leq i \leq t$; $f(u_{3t+i}) = 1 \ \ 1 \leq i \leq t$

$f(u_{4t+i}) = 0 \ \ 1 \leq i \leq t$

Then, $V_f(0) = V_f(1) = V_f(2) = V_f(3) = V_f(4) = t$

$e_f(0) = e_f(1) = e_f(2) = e_f(3) = e_f(4) = t$

Obviously, f is a power 3 Mean cordial labeling.

Case(ii): $n \equiv 1(mod5)$

Let $n = 5t + 1$. Assign labels to the vertices u_i(1 ≤ i ≤ n-1) as in case (i) then assign the label 0 to the vertex U_n

Here $V_f(0) = t + 1$; $V_f(1) = V_f(2) = V_f(3) = V_f(4) = t$

and $e_f(0) = e_f(1) = e_f(2) = e_f(3) = e_f(4) = t$

Obviously, f is a power 3 Mean Cordial labeling.

Case (iii): $n \equiv 2(mod5)$

Now let $n = 5t + 2$, Assign labels to the vertices u_i(1 ≤ i≤ n-1) as in case (ii) then assign the label 1 to the vertex U_n

Here, $V_f(0) = V_f(1) = t + 1$; $V_f(2) = V_f(3) = V_f(4) = t$

and $e_f(0) = e_f(2) = e_f(3) = e_f(4) = t$; $e_f(1) = t + 1$

Obviously, f is a Power 3 Mean Cordial labeling .

Case(iv): $n \equiv 3(mod5)$

Now let $n = 5t + 3$, Assign labels to the vertices u_i(1≤ i ≤ n-1) as in case (iii) then assign the label 2 to the vertex U_n

Here $V_f(0) = V_f(1) = V_f(2) = t + 1$; $V_f(3) = V_f(4) = t$

and $e_f(0) = e_f(3) = e_f(4) = t$; $e_f(1) = e_f(2) = t + 1$

Obviously, f is a Power 3 Mean Cordial labeling .

Case (v): $n \equiv 4(mod5)$

Now let $n = 5t + 4$. Assign labels to the vertices u_i(1 ≤i ≤ n-1) as in case (iv) then assign the label 3 to the vertex U_n

Here $V_f(0) = V_f(1) = V_f(2) = V_f(3) = t + 1$; $V_f(4) = t$

and $e_f(0) = e_f(4) = t$; $e_f(1) = e_f(2) = e_f(3) = t + 1$

Obviously, $f$is a Power 3 Mean Cordial labeling .

From all the above five cases, we conclude that G is a Power 3 Mean Cordial graph.

Example2.2:

Power 3 mean cordial labeling of P_{10} is shown in figure : 2.1

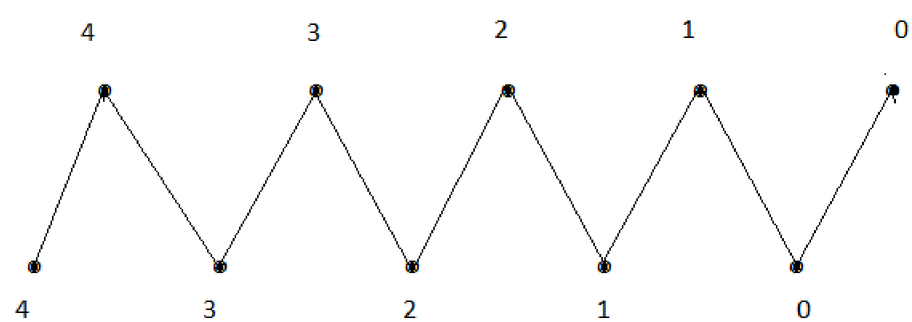

Figure 2.1

Theorem 2.3:

For every n ,comb $P_n \odot K_1$ is a Power 3 Mean Cordial graph.

Proof:

Let P_n be the path $u_1, u_2 \dots \dots u_n$ and let $V(P_n \odot K_1) = V(P_n) \cup \{v_i; 1 \le i \le n\}$

and $E(P_n \odot K_1) = E(P_n) \cup \{u_i v_i; 1 \le i \le n\}$

Case(i): $n \equiv 0(mod 5)$

Let $n = 5t$, Define $f: V(P_n) \to \{0,1,2,3,4\}$

$f(u_i) = 4 \ \ 1 \le i \le t$; $f(u_{t+i}) = 3 \ \ 1 \le i \le t$

$f(u_{2t+i}) = 2 \ \ 1 \le i \le t$; $f(u_{3t+i}) = 1 \ \ 1 \le i \le t$

$f(u_{4t+i}) = 0 \ \ 1 \le i \le t$; $f(v_i) = 4 \ \ 1 \le i \le t$

$f(v_{t+i}) = 3 \ \ 1 \le i \le t$; $f(v_{2t+i}) = 2 \ \ 1 \le i \le t$

$f(u_{3t+i}) = 1 \ \ 1 \le i \le t$; $f(u_{4t+i}) = 0 \ \ 1 \le i \le t$

Then, $V_f(0) = V_f(1) = V_f(2) = V_f(3) = V_f(4) = 2t$

$e_f(0) = 2t - 1$; $e_f(1) = e_f(2) = e_f(3) = e_f(4) = 2t$

Obiviously, f is a power 3 Mean cordial labeling.

Case(ii): $n \equiv 1(mod 5)$

Let $n = 5t + 1$, Assign labels to the vertices u_i and v_i $(1 \le i \le n - 1)$ as in case (i)

Then assign the label 0 and 1 to the vertices u_n , v_n .

Here, $V_f(0) = V_f(1) = 2t + 1$; $V_f(2) = V_f(3) = V_f(4) = 2t$

$e_f(1) = 2t + 1$; $e_f(0) = e_f(2) = e_f(3) = e_f(4) = 2t$

Obiviously, f is a Power 3 Mean Cordial labeling .

Case (iii): $n \equiv 2(mod5)$

Let $n = 5t + 2$. Assign labels to the vertices u_i and v_i $(1 \le i \le n - 2)$ as in case (i) and then assign the label 0,1 and 1,2 to the vertices $u_{n-1}\, u_n, v_{n-1} v_n$ respectively.

Here, $V_f(0) = V_f(2) = 2t + 1$;

$V_f(1) = 2t + 1$; $V_f(3) = V_f(4) = 2t$

$e_f(1) = 2t + 2$; $e_f(0) = e_f(3) = e_f(4) = 2t$; $e_f(2) = 2t + 1$

Obviously, f is a Power 3 Mean Cordial labeling.

Case(iv): $n \equiv 3(mod5)$

Let $n = 5t + 3$. Assign labels to the vertices u_i and $v_i(1 \le i \le n - 3)$ as in case (i) and then assign the label 0,1,2 and 1,2,3 to the vertices u_{n-2}, u_{n-1}, u_n and v_{n-2}, v_{n-1}, v_n respectively.

Here, $V_f(0) = V_f(3) = V_f(4) = 2t + 1$; $V_f(1) = V_f(2) = 2t + 2$

$e_f(0) = e_f(4) = 2t$; $e_f(1) = e_f(2) = 2t + 2$; $e_f(3) = 2t + 1$

Obviously, $f$is a Power 3 Mean Cordial labeling.

Case(v): $n \equiv 4(mod5)$

Let n=5t+4. Assign labels to the vertices u_i and $v_i(1 \le i \le n - 4)$ as in case (i) and then assign the label 0,1,2,3 and 1,2,3,4 to the verticesu_{n-3}, u_{n-2}, u_{n-1}, u_n andv_{n-3}, v_{n-2}, v_{n-1}, v_n respectively.

Here, $V_f(0) = V_f(4) = 2t + 1$; $V_f(1) = V_f(2) = V_f(3) = 2t + 2$

$e_f(0) = 2t$; $e_f(1) = e_f(2) = e_f(3) = 2t + 2$; $e_f(4) = 2t + 1$

Obviously, f is a Power 3 Mean Cordial labeling.

Example 2.4

Power 3 Mean Cordial labeling of $P_{10} \odot K_1$ is shown in figure 2.2

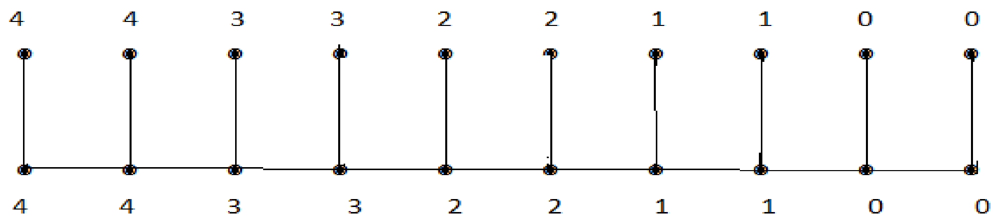

Figure : 2.2

Theorem 2.5

$P_{10} \odot K_1$ is a Power 3 Mean Cordial graph.

Proof:

Let P_n be a path $u_{1,}u_2,\ldots..u_n$. Let $V(P_n \odot K_{1,2}) = V(P_n) \cup \{v_i, w_i;\ 1 \le i \le n\}$

and $E(P_n \odot K_{1,2}) = E(P_n) \cup \{u_i v_i\ u_i\ w_i; 1 \le i \le n\}$

Case (i): $n \equiv 0(mod5)$

Let $n = 5t$, Define $f(u_i) = 4\ \ 1 \le i \le t$; $f(u_{4t+i}) = 0\ \ 1 \le i \le t$

$f(u_{t+i}) = 3\ \ 1 \le i \le t$; $f(u_{2t+i}) = 2\ \ 1 \le i \le t$

$f(u_{3t+i}) = 1\ \ 1 \le i \le t$; $f(u_{4t+i}) = 0\ \ 1 \le i \le t$

$f(v_i) = 4\ \ 1 \le i \le t$; $f(v_{t+i}) = 3\ \ 1 \le i \le t$

$f(v_{2t+i}) = 2\ \ 1 \le i \le t$; $f(u_{3t+i}) = 1\ \ 1 \le i \le t$

Then, $V_f(0) = V_f(1) = V_f(2) = V_f(3) = V_f(4) = 3t$

$e_f(0) = 3t - 1$; $e_f(1) = e_f(2) = e_f(3) = 3t$

Obviously, f is a power 3Mean cordial labeling.

Case(ii): $n \equiv 1(mod5)$

Let $n = 5t + 1$. Assign labels to the vertices u_i , v_i *and* $w_i(1 \leq i \leq n - 1)$ as in case (i).

Then assign the label 0 to the vertices u_n , v_n and 1 to the vertices w_n

Here, $V_f(0) = 3t + 1, V_f(1) = 3t + 1$; $V_f(2) = V_f(3) = V_f(4) = 3t$

$e_f(0) = 3t + 1$; $e_f(1) = 3t + 1$; $e_f(2) = e_f(3) = e_f = 3t$

Obviously, f is a Power 3 Mean Cordial labeling .

Case (iii): $n \equiv 2(mod5)$

Let $n = 5t + 2$, Assign labels to the vertices u_i , v_i *a and* $w_i(1 \leq i \leq n - 2)$ as in case (i) andthen assign the label 0,1 to the vertices $u_{n-1}\, u_n$, *and* $v_{n-1}v_n$ and 1,2 to the vertices w_{n-1}, w_n respectively.

Here, $V_f(0) = 3t + 2$; $V_f(1) = 3t + 3$;

$V_f(2) = 3t + 1$; $V_f(3) = V_f(4) = 3t$

$e_f(1) = 3t + 3$; $e_f(0) = 3t + 1$, $e_f(3) = e_f(4) = 3t$; $e_f(2) = 3t + 1$

Obviously, f is a Power 3 Mean Cordial labeling.

Case(iv): $n \equiv 3(mod5)$

Let $n = 5t + 3$. Assign labels to the vertices u_i , v_i*and* $w_i(1 \leq i \leq n - 3)$ as in case (i) and then assign the label 0,1,2 to the vertices u_{n-2}, u_{n-1}, u_n and v_{n-2}, v_{n-1}, v_n and assign the label 1,2,3 to the vertices w_{n-2}, w_{n-1}, w_n respectively.

Here, $V_f(0) = 3t + 2, V_f(1) = V_f(2) = 3t + 3$

$$V_f(3) = V_f(4) = 3t + 1$$

$e_f(0) = e_f(3) = e_f(4) = 3t + 1$; $e_f(1) = e_f(2) = 3t + 3$

Obviously, f is a Power 3 Mean Cordial labeling.

Case(v): $n \equiv 4(mod5)$

Let $n = 5t + 4$. Assign labels to the vertices u_i, v_i and $w_i(1 \leq i \leq n-4)$ as in case (i) and then assign the label 0,1,2,3 to the vertices$u_{n-3}, u_{n-2}, u_{n-1}, u_n$ and$v_{n-3}, v_{n-2}, v_{n-1}, v_n$ and assign the label 1,2,3,4 to the vertices$w_{n-3}, w_{n-2}, w_{n-1}, w_n$ respectively.

Here, $V_f(0) = 3t + 1$; $V_f(4) = 3t + 1$

$V_f(1) = V_f(2) = 3t + 4$; $V_f(3) = 3t + 1$

$e_f(0) = e_f(4) = 3t + 1$; $e_f(1) = e_f(2) = 3t + 4$; $e_f(3) = 3t + 2$

Obviously, f is a Power 3 Mean Cordial labeling.

Example 2.4 Power 3 Mean Cordial labeling of $P_8 \odot K_{1,2}$ is shown below

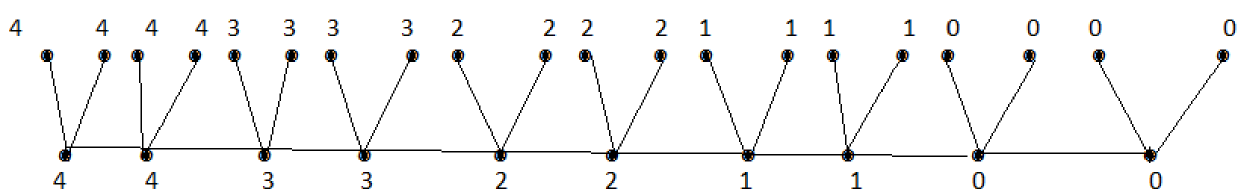

Figure 2.3

3.Conclusion:

In this paper ,we studied the Power 3 Mean Cordial labeling of some graphs. It is very interesting to investigate graphs which admit Power 3 Mean Cordial Graphs. In this paper we proved some path ,comb , $P_8 \odot K_{1,2}$ are Power 3 Mean Cordial Graphs. It is demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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References:

[1] Albert Wiiliam ,Indra Rajasingh and S.Roy ,Mean Cordial Labeling of certain Graphs ,Journal of computer and Mathematical Sciences ,4(2013),no.3,274-281.

[2] F.Harary ,Graph Theory ,Narosa Publishing House ,New Delhi,1988.

[3] J.A.Gallian ,A Dynamic Survey of Graph Labeling ,The Electronic Journal Combinatories, (2013).

[4] Raja Ponraj ,Muthirulan Sivakumar ,Murugesan Sundraram, Mean Cordial Labeling of Graphs, Open Journal of Discrete Mathematics, 2(2012),145-148 https://doi.org/10.4236/ojdm.2012.24029

[5] S.S.Sandhya, S.Sreeji Power 3 Mean Labeling of Graphs, International Journal of Mathematical Analysis Vol. 14, 2020, no.2, 51 – 59 https://doi.org/10.12988/ijma.2020.91271