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Oscillatory Solutions of Third Order nonlinear Neutral difference Equation with Delay

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Abstract

In this article, we study the nonlinear neutral delay difference equation

$$\Delta\left(\frac{1}{r_{2l}}\,\Delta\left(\frac{1}{r_{1l}}\Delta(y_l+p_l y_{l-\tau})\right)\right)+f_l\big(g(y_{l-\sigma})\big)=0 \quad , (1.1)$$

where the sequence {y_l } of real numbers, and r_2l,r_1l be a positive real sequence, and p_l,f_l be a positive sequence. we establish new oscillation conditions for all solutions of neutral delay difference equations.

Keywords: Third – order, oscillatory solutions, delay difference equations.
2010 MSC: 39A10

INTRODUCTION

In this paper, we shall consider the nonlinear neutral delay difference equation

$$\Delta\left(\frac{1}{r_{2l}}\,\Delta\left(\frac{1}{r_{1l}}\Delta(y_l+p_l y_{l-\tau})\right)\right)+f_l\big(g(y_{l-\sigma})\big)=0 \quad (1.1)$$

where the sequence $\{y_l\}$ of real numbers, and r_{2l},r_{1l} be a positive real sequences, and p_l,f_l be a positive sequence, and Δ indicate that the forward difference operator by $\Delta y_n = y_{n+1}-y_n$
wedefine $\sum_{l=l_0}^{l} r_{1l} = \sum_{l=l_0}^{l} r_{2l} = \infty,\ l_0 \geq l$.

In recent years, there has been an increasing interest in the study of the oscillatory behavior of third order difference equations, see for example [12-17,23,24]. For general theory of oscillation of difference equations, we refer to [1-6,10,11];
Some recent results on third order difference equations can be seen [7-9,18-22], [25-29].
In the equation (1.1) if p_l=0

$$\Delta\left(\frac{1}{r_{2l}}\,\Delta\left(\frac{1}{r_{1l}}\Delta(y_l)\right)\right)+f_l\big(g(y_{l-\sigma})\big)=0 \quad ,$$

which is a delay equation and further if we take P_1=0 and σ=0 in equation (1.1) ,

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We get

$$\Delta\left(\frac{1}{r_{2l}}\,\Delta\left(\frac{1}{r_{1l}}\Delta(y_l)\right)\right)+f_l(g(y_l))=0 \quad (1.3)$$

Which is an ordinary difference equation. Now, we see some special case of equation (1.1)

When
R_{2l}=1 and R_{1l}=1 and g(Y_l)=Y equation (1.1) is reduced to

$$\Delta^3(y_l+p_l y_{l-\tau})+f_l(y_l)=0 \quad (1.4)$$

If further when P_l=0, this equation reduced by

$$\Delta^3(y_l)+f_l(y_l)=0. \quad (1.5)$$

A solution of equation (1.1) is called oscillatory if it has arbitrary large zeros on [Y_l,∞) ; otherwise it is called non oscillatory. Equation (1.1) is said to be oscillatory it all its solutions oscillate. Unless otherwise stated, when we write a functional inequality, it will be assumed to held for sufficiently large l in our subsequent discussion.

2.Main Results

We need the following in our discussion.

(f_1): r_{1l}, r_{2l} are positive real sequences;
(f_2): f_l is non decreasing real sequence;
(f_3): p_l is sequence of positive with $0< p_l < 1$;
(f_4): g: R→R is continuous function such that x g(x)>0 for x≠0;
(f_5): There exist, q >0, s.t $f_l g(x) \geq q_l(x)$;
(f_6): $\sum_{l=l_0}^{\infty}\frac{1}{r_{1l}}=\infty$ and $\sum_{l=l_0}^{\infty}\frac{1}{r_{2l}}=\infty$.

We set

$$z_l = y_l + p_l y_{l-\tau}$$

Lemma 2.1:
Let (F_1) and (F_4) hold. Let Y_l be a positive solution of (1.1) and suppose that,
Case (I):
$z_l > 0$, $\Delta z_l > 0$, $\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right) > 0$ hold . Then there exists $l_1 \geq l_0$ sufficiently large,

Such that,

$$\frac{1}{r_{l-\sigma}}\Delta z_{l-\sigma} \geq \delta_{l-\sigma}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right) \text{ for } l_1 \geq l_0 \quad (2.1)$$

Let D= (m, n) be a positive and {H(mn):m≥n≥0} be a double sequence such that

i) $H_{m,m} = 0$ for m≥ 0;
ii) $H_{m,n} > 0$ for m > n ≥ 0;
iii) $\Delta_2 H_{m,n} = H_{m,n+1} - H_{m,n} \leq 0$; and

$$\Delta_2 H_{m,n} = -h_{m,n}\sqrt{H_{m,n}} \text{ for } m\geq n\geq 0.$$

Theorem:2.1

Assume that (F_1) – (F_4) hold. Let Pl be a positive real sequence and Let H: D→R be real continuous functions, such that H belongs to the class F and suppose that there exists an interval (a, b)⊂ [L0,∞) such that p_l⩾0 and f_l>0 , for all lϵ(a,b) and there exists c ϵ(a,b) and ρϵc'([L(0,) ∞),(o,∞)) such that

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$$\frac{1}{H(c,a)}\sum_{n=m}^{c} H(n,a)\rho_{l+1}q_l[1-p_{l-\sigma}] + \frac{1}{H(b,c)}\sum_{n=c}^{m} H(b,n)\rho_{l+1}q_l[1-p_{l-\sigma}] > \frac{1}{4}\left\{\frac{1}{H(c,a)}\sum_{n=m}^{c} g_l h_1(c,a)\sqrt{1+(c,a)}\,\rho_l + H(c,a)\Delta^2\rho_l + \frac{1}{H(b,c)}\sum_{n=c}^{m} g_l h_2(b,n)\sqrt{H(b,c)} - H(b,c)\,\Delta^2\rho_l\right\} \quad (2.2)$$

were $g_l = \frac{1}{\delta_{l-\sigma} r_{l-\sigma}\rho_l}$.

Then (1.1) is oscillatory.

Proof: Suppose to the contrary that y_l is a non oscillatory solution of the equation (1.1). without loss of generality, we may assume that y_l is an eventually positive, since the case when y_l is negative can be dealt similarly

We set $z_l = y_l + p_l y_{l-\tau}$

Hence, from (f_5) equation (1.1) becomes.

$$\Delta\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta(y_l + p_l y_{l-\tau})\right)\right) \le -q_l(y_{l-\tau}) \quad (2.3)$$

Further we obtain;

$$z_l = y_l + p_l y_{l-\tau}$$
$$y_l = z_l - p_l y_{l-\tau}$$
$$y_{l-\sigma} = z_{l-\sigma} - p_{l-\sigma} y_{l-\sigma-\tau},$$
$$y_{l-\sigma} \ge 1 - p_{l-\sigma} z_{l-\sigma} \quad (2.4)$$

Hence from (2.3) and (2.4) we have,

$$\Delta\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta(y_l + p_l y_{l-\tau})\right)\right) \le -q_l[1-p_{l-\sigma}]\, z_{l-\sigma} \quad (2.5)$$

Define;

$$\psi_l = \rho_l \frac{\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)}{z_{l-\sigma}} \quad (2.6)$$

$$\Delta\psi_l = \Delta\rho_l\left[\frac{\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)}{z_{l-\sigma}}\right] + \rho_{l+1}\left[\frac{\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)}{z_{l-\sigma}}\right]$$

$$\Delta\psi_l = \Delta\rho_l\left[\frac{\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)}{z_{l-\sigma}}\right] + \rho_{l+1}\left[\frac{z_{l-\sigma}\Delta\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right) - \left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)\Delta z_{l-\sigma}\right)}{z_{l-\sigma}z_{l-\sigma+1}}\right]$$

$$\Delta\psi_l = \Delta\rho_l\left[\frac{\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)}{z_{l-\sigma}}\right] + \rho_{l+1}\left[\frac{\Delta\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)}{z_{l-\sigma+1}}\right] - \rho_{l+1}\left[\frac{\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)\Delta z_{l-\sigma}}{z_{l-\sigma}z_{l-\sigma+1}}\right]$$

$$\Delta\psi_l = \frac{\Delta\rho_l}{\rho_l}\left[\rho_l\frac{\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)}{z_{l-\sigma}}\right] + \rho_{l+1}\left[\frac{\Delta\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)}{z_{l-\sigma+1}}\right] - \rho_{l+1}\left[\frac{\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)\Delta z_{l-\sigma}}{z_{l-\sigma}z_{l-\sigma+1}}\right]$$

$$\Delta\psi_l = \frac{\Delta\rho_l}{\rho_l}\psi_l + \rho_{l+1}\left[\frac{\Delta\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)}{z_{l-\sigma+1}}\right] - \rho_{l+1}\left[\frac{\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)\Delta z_{l-\sigma}}{z_{l-\sigma}z_{l-\sigma+1}}\right]$$

$$\Delta\psi_l = \frac{\Delta\rho_l}{\rho_l}\psi_l - \rho_{l+1}q_l[1-p_{l-\sigma}] - \rho_{l+1}\left[\frac{\left(\frac{1}{r_{2l}}\Delta\left(\frac{1}{r_{1l}}\Delta z_l\right)\right)\Delta z_{l-\sigma}}{z_{l-\sigma}z_{l-\sigma+1}}\right]$$

In equation (2.1) we get;

$$\Delta\psi_l \le \frac{\Delta\rho_l}{\rho_l}\psi_l - \rho_{l+1}q_l[1-p_{l-\sigma}] - \rho_{l+1}\left[\frac{\psi_l}{\rho_l}\,\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{r_l z_{l-\sigma}}\,\Delta z_l\right]$$

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$$\Delta\psi_l \le \frac{\Delta\rho_l}{\rho_l}\psi_l - \rho_{l+1}q_l[1-p_{l-\sigma}] - \frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2$$

$$\Delta\psi_l \le -\rho_{l+1}q_l[1-p_{l-\sigma}] + \frac{\Delta\rho_l}{\rho_l}\psi_l - \frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2$$
(2.7)

Multiplying the above inequality by H(m,n),

We get,

H(m,n)
$$\Delta\psi_l \le -H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] + H(m,n)\frac{\Delta\rho_l}{\rho_l}\psi_l - H(m,n)\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2$$

$$H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le -\mathrm{H(m,n)}\, \Delta\psi_l + H(m,n)\frac{\Delta\rho_l}{\rho_l}\psi_l - H(m,n)\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2$$
(2.9)

Summing by parts formula the above inequality we have

$$\sum_{n=c}^{m} H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le -\mathrm{H\ (m,m)}\psi_l + \mathrm{H\ (m,c)}\psi_l + \sum_{n=c}^{m}\Delta_2 H(m,n)\psi_{l+1} + \sum_{n=c}^{m} H(m,n)\frac{\Delta\rho_l}{\rho_l}\psi_l - \sum_{n=c}^{m} H(m,n)\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2$$
(2.10)

Since H(m,m)=0 ,we obtain
$$\sum_{n=c}^{m} H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le \mathrm{H\ (m,c)}\psi_l + \sum_{n=c}^{m}\Delta_2 H(m,n)\psi_{l+1} + \sum_{n=c}^{m} H(m,n)\frac{\Delta\rho_l}{\rho_l}\psi_l - \sum_{n=c}^{m} H(m,n)\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2$$
(2.11)

$$\sum_{n=c}^{m} H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le \mathrm{H\ (m,c)}\psi_l - \sum_{n=c}^{m} H(m,n)\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2 \quad \sum_{n=c}^{m}\Delta_2 H(m,n)\psi_{l+1} + \sum_{n=c}^{m} H(m,n)\frac{\Delta\rho_l}{\rho_l}\psi_l$$

$$\sum_{n=c}^{m} H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le \mathrm{H\ (m,c)}\psi_l - \sum_{n=c}^{m} H(m,n)\frac{\delta_{l-\sigma}\, r_{l-\sigma}}{\rho_l}{\psi_l}^2 + \sum_{n=c}^{m}\left\{h(m,n)\sqrt{H(m,n)} - H(m,n)\frac{\Delta\rho_l}{\rho_l}\right\}\psi_l$$

$$\sum_{n=c}^{m} H(m,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le \mathrm{H\ (m,c)}\psi_l + \frac{1}{4}\sum_{n=c}^{m}\frac{1}{H(m,n)\,\delta_{l-\sigma}\, r_{l-\sigma}\rho_l}\, .\, h_1(m,n)\sqrt{H(m,n)}\rho_l + H(m,n)\Delta^2\rho_l$$

Similarly multiplying (2.7) by H (m, n) and summing from m to c-1 for m∊(a,c] one obtains

$$\sum_{n=m}^{c} H(n,m)\rho_{l+1}q_l[1-p_{l-\sigma}] \le \mathrm{H\ (c,m)}\psi_l + \frac{1}{4}\sum_{n=m}^{c}\frac{1}{H(c,n)\,\delta_{l-\sigma}\, r_{l-\sigma}\rho_l}\, h_2(n,m)\sqrt{H(n,m)}\rho_l - H(n,m)\Delta^2\rho_l$$

Letting m→a+ in (2.14) and m→b- in (2.13). dividing (2.13) & (2.14) by H (b, c) and H (c, a) respectively and adding them, we have that

$$\frac{1}{H(c,a)}\sum_{n=m}^{c} H(n,a)\rho_{l+1}q_l[1-p_{l-\sigma}] + \frac{1}{H(b,c)}\sum_{n=c}^{m} H(b,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \le \frac{1}{4}\left\{\frac{1}{H(c,a)}\sum_{n=m}^{c}\frac{1}{\delta_{l-\sigma}\, r_{l-\sigma}\rho_l}\, .\, h_1(c,a)\sqrt{H(c,a)}\rho_l + H(c,a)\Delta^2\rho_l + \frac{1}{H(b,c)}\sum_{n=c}^{m}\frac{1}{\delta_{l-\sigma}\, r_{l-\sigma}\rho_l}\, .\, h_2(b,c)\sqrt{H(b,c)}\rho_l - H(b,c)\Delta^2\rho_l\right\}$$

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$$\begin{aligned}
&\frac{1}{H(c,a)}\sum_{n=m}^{c} H(n,a)\rho_{l+1}q_l[1-p_{l-\sigma}] \\
&\quad +\frac{1}{H(b,c)}\sum_{n=c}^{m} H(b,n)\rho_{l+1}q_l[1-p_{l-\sigma}] \\
&\quad \le \frac{1}{4}\left\{\frac{1}{H(c,a)}\sum_{n=m}^{c} g_l\ .h_1(c,a)\sqrt{H(c,a)}\rho_l + H(c,a)\Delta^2\rho_l\right. \\
&\quad \left. +\frac{1}{H(b,c)}\sum_{n=c}^{m} g_l\ .h_2(b,c)\sqrt{H(b,c)}\rho_l - H(b,c)\Delta^2\rho_l\right\} \qquad (2.15)
\end{aligned}$$

where $g_l = \frac{1}{\delta_{l-\sigma}\, r_{l-\sigma}\rho_l}$

This contradicts the assumption (2.2), The proof is complete

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